

ABSTRACT

Title of Dissertation: UNDERSTANDING OPPORTUNITIES TO PRACTICE
WHAT WE PREACH: MATHEMATICAL
EXPERIENCES OF MATHEMATICS EDUCATION
DOCTORAL STUDENTS

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Often times, there is a disconnect between the way in which mathematics education courses suggest the teaching and learning of mathematics should occur and the way mathematics education doctoral students actually experience mathematics teaching and learning. By asking mathematics education doctoral students to reflect on their mathematical experiences, I was able to further understand the nature of and impact of these experiences. In this study, I engaged in conversations with five other mathematics education doctoral students asking them to share their mathematics experiences in their doctoral preparation. Several factors influenced doctoral student mathematical experiences such as their perceptions of the nature of authority in the classroom, the level of interaction (between student and instructor, among students, and between the students and the mathematics), and nature and purpose of the mathematics in the course.

The most typical experiences that doctoral students identified were consistent with traditional lecture courses where the instructor deposited knowledge upon students. Several participants reported on courses that broke the traditional mode and offered some authority to students in the decision-making in the classroom or encouraged a higher level of interaction by engaging students in group work. One particular set of courses deemed “influential”, “transformational”, and “empowering” embodied a classroom where inquiry reigned and the course environment allowed, required, and supported students to explore their own mathematical questions. This special set of courses featured a shared authority between instructors and students, engaged students in a high level of interaction, and explored mathematical questions and problems that came from the students’ questions and ideas rather than from a textbook. The purpose of the course was a place where students could have an authentic opportunity to *do* mathematics. The importance of the mathematical inquiry in these courses, according to participants, was not only important for themselves as learners of mathematics but also necessary for their preparation of becoming mathematics educators.

UNDERSTANDING OPPORTUNITIES TO PRACTICE WHAT WE PREACH:
MATHEMATICAL EXPERIENCES
OF MATHEMATICS EDUCATION DOCTORAL STUDENTS

by

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2008

Dedication

This work is dedicated to:

My parents, Donna and Ljubomir Vuchichevich, with profound admiration and gratitude.
Thank you for your endless support and belief in me.

Mom,

As a teacher by trade, you taught me to love learning and teaching. As a mom, you have been a model of strength and love that I will always aspire to. Thank you for all you have done and continue to do for me. I could not have accomplished this without you. I love you!

Tata,

The wisdom and love of life you left behind continues to fill my soul. Your gifts of love and laughter have helped me in countless ways on this journey. *Dragi Moj Tata, Volim te, nedostajes mi. Do naseg vidjenja u kuhinji...Ana*

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Prologue

This research is about people's experiences in the world. The method of seeking understanding of experience should reflect the humanness in that endeavor. Donald Polkinghorne has argued that research as a human practice approach should, "capture the human actions and temporal character of the research process," (Polkinghorne, 1997). He makes a distinction between a humanistic approach to conducting and reporting research and the traditional reporting of research that is strongly influenced by the scientific method and is rigidly structured in a doctoral theses as: Introduction, Literature Review, Methodology, Result, Conclusions, and Implications. My aim to stay true to myself while presenting the following as scholarly research manifests in the form that this work is presented. The presentation of study is guided by the traditional structure of a doctoral theses but is supplemented by interludes and vignettes, that I believe help capture the essence of the work.

This dissertation has been a professional and personal journey of discovery. Its contents and form reveal a great deal of my identity as a writer, researcher, teacher, and knower and doer of mathematics. I find it a mistake to detach who I am from what I have come to know through this journey of understanding the mathematical experiences of mathematics education doctoral students. To that end, I have chosen to include a river of mathematical ideas that have developed in me from the time this project began, which without hesitation I believe began when I was a young student of mathematics, to its relative endpoint- the completion of this project. Throughout the dissertation, I have included a variety of writings that highlight mathematical questions, problems, understanding and ideas that I grappled with, hated, loved, and came to know. I believe

that it is in the public sharing of these ideas that help others connect what it is I experienced on my journey with those on similar journeys before, during, and after mine. It is these connections that help to situate the experiences in the larger arena of doctoral preparation in mathematics education.

These mathematical interludes are presented chronologically. They are not meant to be a continuation of each other, nor do they put together a specific piece of mathematics. They do however, show to the reader a way in which my mathematical-ness evolved throughout the journey of being a mathematics education doctoral student. Some interludes are mostly mathematical in nature and present few of my own reflections, while others are mostly reflective.

Interlude One is a return to a mathematical event that occurred during my first year of teaching. It describes an investigation that I engaged in while a student in a mathematics course designed for mathematics education doctoral students. It shows the result of the opportunity to go back and explore a mathematical topic that was of interest/problematic in my past.

Interlude Two is a reflection that I wrote a shortly after being in a mathematical discussion with others finding that many mathematicians believe that $.9$ repeating is equal to 1. This particular topic was a reoccurring topic of discussion in the aforementioned course.

Interlude Three was written after a day at an institute designed to help mathematics educators plan and teach courses such as one that inspired this dissertation. It was the result of an afternoon of work on questions that *I* was curious about and shows my mathematical ability and determination to share it with others.

It is my desire that through these interludes the reader will get a sense of how my mathematical course experiences have affected my evolving notion of how and why I, a mathematics educator, have become intimately concerned with the knowing and doing of mathematics.

CHAPTER ONE

This chapter is organized into two sections. The first section introduces the study and provides an overview of my mathematical experiences and preparation that have influenced my coming to this study. The second section describes the importance of this study and includes a brief description of how this study fits into the broader arena of mathematics education.

Section One: Introduction To A Problematic Situation

Preparing for a life in academia is supposed to occur in university doctoral programs. In many cases, there seems to be a strong bias toward preparing doctoral candidates for careers in academia, particular at research-intensive universities. But even in these instances, the only preparation for academic careers that occurs during the doctoral education is likely to be training in research (LaPidus, 1997). One could argue that a crucial criterion for obtaining a Ph.D. is the ability to design and conduct an independent research project, or doctoral dissertation. This focus on knowing and doing research very often permeates the content of doctoral courses, assignments, and out-of-class experiences and collaboration. Prospective employers of new Ph.D.'s, however, are looking to hire candidates with an increasingly diverse set of skills, experiences, and knowledge (Fey, 2001; LaPidus, 1997).

In my own experiences as a mathematics education doctoral student, I felt the pressure of learning about and practicing research while trying to prepare for my future. As graduation becomes an increasingly brighter light at the end of the tunnel, questions about my preparedness as an employee become clear. What is it that we, as mathematics

education doctoral students, are supposed to know and be able to do when we graduate?

What kinds of experiences, knowledge, and dispositions are potential employers requiring? The most prestigious positions, tenure-track faculty positions, list a slew of qualifications. On a recent job search for mathematics education faculty positions on the Internet, I found most job postings included a laundry list of applicant qualifications and job duties. A composite of these qualifications reads:

All candidates must have:

- an earned doctorate in mathematics education or a closely related field, or a doctorate in mathematics with course work and experience in mathematics education;
- a strong preparation in mathematics (frequently 18 hours of graduate level mathematics);
- a research focus in mathematics education (including a broad range of research interests in mathematics teaching and learning);
- a record of publication commensurate with their intended rank;
- teaching, mentoring, and supervising experience with pre-service or in-service teachers; and
- knowledge of standards, theory, and research in mathematics.

Preferred candidates are individuals with:

- experience teaching in K-12 settings;
- commitment to urban education, including either experience in urban settings or course work in urban education;

- curriculum development experience at the elementary, middle or secondary level;
- facility with the use of technology in teaching mathematics;
- experience in statistics and statistical methods; and
- leadership experience as a cooperating teacher, and/or staff developer.

This is a list of qualifications exclusively for university positions. A different list of qualifications and experiences exists for mathematics education doctoral candidates choosing to work in school districts, departments of education, curriculum development, professional development, or K-12 classrooms. Such extensive lists indicate the need of multi-faceted coursework and rich experiences during the mathematics education doctoral preparation.

In mathematics education an added layer seems to complicate the matter. During our doctoral studies and training, we study mathematics, mathematics education, how students learn mathematics, where mathematical knowledge comes from, and proposed best practice in mathematics teaching. In my own experiences as a doctoral student, this also included learning about and researching the kinds of mathematical activities suggested by the National Council of Teachers of Mathematics (NCTM) as best for learners of mathematics. Specifically, we have had opportunities to study the goals and research supporting the NCTM's (2000) Principles and Standards for School Mathematics document.

Beyond studying and researching these "Standards-based" ideas and convictions, we are asked to pass this knowledge on to those whom we teach both in what we teach and how we teach it. This includes undergraduate and graduate mathematics methods

courses, mathematics for teaching courses, and professional development for inservice teachers. We are asked to learn about *it* and teach *it*, but in most cases, we, as doctoral students, have never experienced what *it* is that we have been learning and teaching. Our experiences in mathematics as K-12 students occurred well before the Standards movement, as we know it today. Our college and graduate school experiences in mathematics followed the large lecture format for which mathematics departments are known. These most recent experiences as math students epitomized a traditional teacher-as-all-knowing or recitation style of teaching and learning.

Conversations have occurred in the field of mathematics education regarding the importance of a variety and intensiveness of experiences that doctoral students have in the span of their program (Reys & Kilpatrick, 2000). The National Science Foundation has recently funded two National Conferences on Doctoral Programs in Mathematics Education. These conferences provide a place to examine, discuss, and make recommendations for the future of doctoral programs in mathematics education. The conversations tend to focus on the kinds of experiences that students have outside of coursework. Blume (2001) described non-course experiences (NCE's) as activities such as teaching, engaging in research, planning and delivering teacher professional development, and preparing articles for submission to professional journals. It is assumed that engagement in these types of activities will help mathematics education doctoral students become prepared for their careers after graduating.

These conversations have only briefly focused on the kinds of course experiences mathematics education doctoral students should have. It may be typical to discuss the amount or types of mathematics courses doctoral students should take, but the kinds of

mathematical experiences that mathematics education doctoral students should be having is seldom discussed. When mentioned, the content of mathematics learning during mathematics education doctoral students' programs was described as "torturous" (Fennell, Briars, Crites, Gay & Tunis, 2001). Indeed, the participants agreed that mathematics content courses were vital to a mathematics education doctoral program. Many acknowledged, "Ideally, mathematics experiences should reflect the most appropriate pedagogy..." (Fennell, Briars, Crites, Gay & Tunis, 2001, p. 41). Participants concluded that continued dialogue and inquiry were needed to improve the quality of doctoral programs in mathematics education.

My own mathematics education doctoral experiences occurred while being a National Science Foundation (NSF) fellow in the Mid-Atlantic Center for Mathematics Teaching and Learning (MAC-MTL), a center originally charged to

- conduct research on the development, application, and influence of teacher knowledge in K-12 mathematics;
- develop models for the mathematical education of pre-service teachers and the professional development of in-service mathematics teachers; and
- design and operate collaborative programs of doctoral and postdoctoral studies that prepare future leaders for mathematics education (MAC-MTL, n.d.).

In the center's original NSF proposal request, MAC-MTL claimed, "A strong program for doctoral specialists will be characterized by these major elements:

- Breadth and depth of knowledge in mathematics and its applications.
- Understanding of learning contexts.
- Scholarly skills" (MAC-MTL, n.d.).

The proposal explicitly describes what “breadth and depth of knowledge in mathematics and its applications” encompasses:

This knowledge emphasizes a connected understanding of the structural underpinnings of mathematics and its applications—a perspective that goes beyond understanding and technical skill in particular topics to include awareness of mathematical habits of mind, historical development of big ideas and the contributions of different cultures, personal confidence in one’s ability to do and learn mathematics, and a disposition to keep growing mathematically (MAC-MTL, n.d.).

The center’s proposal went on to describe the kinds of mathematics experiences that would support students in gaining “breadth and depth of knowledge in mathematics and its applications”. The center proposed a series of mathematics courses, Foundations of Mathematics I and II, that would by design provide students with rich experiences, and that would “begin with questions at the heart of school mathematics and develop connections to important topics at other levels and in other branches of the subject”. The development of the course was a challenge for the center, as suggested by the numerous iterations in course design, structure, and instructors.

And so I ask, when do we, as future mathematics educators, have the opportunity to be learners in rich, authentic mathematical situations? When do we have the opportunity to seek answers to our mathematical questions? The theories, standards, and methods that we stand behind and claim to believe in have almost never been experienced by contemporary mathematics education students. We advocate that students should experience rich, authentic, and meaningful mathematics. However, I posit that it is rare to encounter mathematics education doctoral students who have experienced learning mathematics in this way. This dichotomy has provided the backdrop to my research.

A Look Inward at the Problem

As researcher, I do not claim to engage in this human-to-human activity without acknowledging my own humanness. Through this research, I hope to better understand myself, my experiences, and the experiences of those around me. I am a mother, daughter, wife, and friend. By profession and passion, I am a teacher and a student. In addition to the personal characteristics, perspectives, and questions that define me, I carry with me the “complex and contradictory history” of qualitative research perspectives on this learning journey (Denzin & Lincoln, 2005). As I ask, seek, and write to understand, I am constantly confronted with ethics and politics of such research. (Denzin & Lincoln, 2005).

My own doctoral preparation has included a myriad of experiences. I have taken courses in foundations of education, research methodology, mathematics, and mathematics education. The objectives behind the mathematics education coursework were to, “examine issues, historical experiences, emerging trends, theory, and research bases for practice in all aspects of mathematics curriculum, teaching, learning, assessment, policy-making, and professional leadership (MAC-MTL, n.d.)”. My mathematical experiences included taking courses in calculus (two semesters) and abstract algebra. In addition, I took courses entitled, “Transition to Advanced Mathematics” and “Researching Mathematics as Mathematics Educators” (two semesters).

I look to the experiences of others walking near me on this journey to help me ground my own experiences. I use the experiences of others, not as a comparison or contrast to my own, but rather, as a bigger picture of what it is I am seeking to understand

as I haven't experienced my doctoral education in a vacuum. My existence and experiences in a doctoral program have influenced others', and they in turn, have influenced mine. It is as if my piece of the puzzle, with its twisted shape, cannot be fully understood without also understanding the pieces around me or the big picture of the puzzle. As van Manen (1990) suggests, "One's own experiences are the possible experiences of others and also the experience of others are the possible experiences of oneself" (p. 58). I must consider my own voice and the voice of those around me.

Mathematical experiences from childhood to adulthood.

One of the big misapprehensions about mathematics that we perpetrate in our classrooms is that the teacher always seems to know the answer to any problem that is discussed. This gives students the idea that there is a book somewhere with all the right answers to all of the interesting questions, and that teachers know those answers. And if one could get hold of the book, one would have everything settled. That's so unlike the true nature of mathematics. (Henkin, as cited in L.A. Steen and D.J. Albers (Eds.), 1981).

Mathematics always has been a subject that I was learning *from* my teachers. I close my eyes now and I return to the experiences of learning mathematics. I feel what it was like to sit in a mathematics class. I only now suspect that my senses have been repeatedly cheated in these classes. To really know well or understand deeply, the spectrum of human senses must be engaged. But in my mathematics classes, I was only allowed to hear and to see. The absence of the remaining senses hindered my opportunity to understand the essence and beauty of mathematics.

I did not speak, for my voice was mute, little and insignificant. I was not encouraged to speak freely. The teacher spoke. The teacher asked questions. If I was interested, intrigued or confused about the mathematics and if I had the courage to

question such mathematical tradition, I might have bothered to raise my hand to speak. Consequently, most of the time, I was silent.

I did not breathe in the mathematics and smell the world of number, pattern, and shape. I was not allowed to smell the possibilities or dangers of my mathematical choices and questions. I did not feel, touch or engage with the mathematics or my fellow classmates. The mathematics existed because the teacher told me so, and the book was evidence of its existence. I did not experience what it felt like to travel the slope of a line or walk the perimeter of a polygon. I knew how to calculate these things, but I did not know them.

I experienced mathematics in a way that most people would claim as “ordinary”.

Davis and Hersh (1981) describe an "ordinary mathematics class" as routine:

The program is fairly clearcut. We have problems to solve, or a method of calculation to explain, or a theorem to prove. The main work will be done in writing, usually on the blackboard. If the problems are solved, the theorems proved, or the calculations completed, then the teacher and the class know they have completed the daily task. (p. 3)

This “ordinary-ness” contains a great assumption that the learner will be able to put together the skills and knowledge to understand or produce the whole. In this ordinariness, the teacher is the knower of all things. There are few discussions. Deviations from the norm are considered “not ordinary”, and even detrimental to the learning process. An ordinary teacher tells us, the students, what to do, and we are expected to do it. The line of discourse is almost always one-way. The teacher talks and students respond. Students rarely speak to each other about the mathematics. In this model, there is an assumption that students will absorb and process the incoming information, generalize it, and make sense of it on their own. Some students may learn in

this way. However, research indicates that many students do not (National Research Council, 2001; NCTM, 2000).

What are some of the things I had become accustomed to in ordinary mathematics classes? I had gotten used to not really knowing. It no longer bothered me that I did not really understand. I got used to not asking questions. I got used to being apprehensive in my curiosity. I got used to being quiet both physically and mentally. It was not worth the bother anymore since I was getting good grades in ordinary mathematics class. In this excerpt from a personal journal, I recall an instance in high school that began the unveiling of my not-knowing.

I find myself sitting on the outside of the mathematics club - a club that I had once ironically belonged to in high school. I was one of them. I had been asked to join the math team for the conference meet. Turns out someone was injured - I was asked to suit up for the big game. I must have been good enough for the team picture. My memories around the math meet are bittersweet. I remember the pre-game jitters, the game plan we went over on the bus - answer the questions you think you know first then go back and tackle the ones that are harder. I followed the game plan. Except after taking a pass through the test, I had hardly answered any questions. I was cold, frozen and not performing at game time. I could hear the silent screams of my coach and teammates. They all appeared to be furiously writing away and I sat and looked at my paper. How did I end up here? I was in the fast track in mathematics for most of my life. But why? I don't have vivid memories of enjoying mathematics particularly. I was successful at getting good grades. But the mathematics club? The best math classes were the ones where the teacher let you know ahead of time that the homework wouldn't really be "corrected" by the teacher. Instead, the teacher would quickly go around the room and glance at your notebook and you would get full credit for attempting the problems and having some kind of answer for each problem. And then the best part - the teacher would go over all the answers and students could "fix" any problems that were incorrect and we could spend time going over particularly difficult problems. If my teachers really knew how little mathematics I had been able to do on my own... I used the answers provided to memorize problems and steps to help me get good enough grades on tests. I was able to keep up with A's until about calculus. Then things got tricky. I rationalized getting B's because it was calculus and I accepted the muddled understanding because it was calculus and who really understood what was going on anyway.

The problem I still remember. What two numbers, that happen to be palindromes are two minutes away from each other? I sat in the room and stared

at the clock (which didn't help all that much because it was analog) but I was staring at the clock anyway so... it seemed so easy. But I couldn't think of it. My mind raced. How could I be on the math team and not be able to make this easy shot? It should be fairly obvious, should it not? Should I be spending time on this problem? Considering the other problems appeared to be much more involved, I decided to stick with this problem - but was it a waste of time? I mean I could actually find the answer by running through every single set of minutes so I might as well. I spent the remainder of the test writing out pairs of numbers - I had started at 12:00. Unfortunately, the palindrome doesn't appear until much later in the day - and the referee blew his whistle before I could get the answer.

Although you will find me in the mathematics club yearbook picture for that year, I was not asked to play in another game or try out for the team the following year. It was during this pivotal moment that I became aware of my own unknowing in a new way. It was somewhat shocking, almost embarrassing. How had I gotten to this place?

The bothering question.

After completing my undergraduate degree in Curriculum and Instruction, I began teaching in a large urban school district in the Midwest. It was during this time that my curiosity about the teaching and learning of mathematics was awakened. I was hired at a school that was piloting a reform-based mathematics curriculum. As a new teacher, I was required to attend professional development to become familiar with the goals, objectives and philosophy of the curriculum. Unlike any mathematics curriculum that I had ever experienced, I was quickly fascinated by what was going on mathematically in my classroom. I had to learn more.

Six months after I began teaching, I enrolled in a Master's program with a focus in elementary mathematics education. At the same time, I began to become heavily involved in the district's mathematics professional development team. I enjoyed sharing what I was learning in my Master's program as well as what was occurring in my third grade classroom with other teachers in the district. Within weeks of graduation, my

advisor suggested I apply to a newly designed mathematics education doctoral program sponsored by the National Science Foundation.

In the fall of 2001, I began my doctoral program at a large university in the Mid-Atlantic region of the United States. My acceptance was provisional at first until I was able to successfully complete a two semesters of undergraduate Calculus. I had taken Pre-Calculus and A. P. Calculus in high school as well as a semester of Calculus in my undergraduate program, but the faculty felt the Calculus course would be a good mathematical starting point for me in my doctoral experience. It was here, in my doctoral program, that my bothering about mathematical questions was pushed and pulled in ways I could not foresee. It was here that I felt increasingly bothered about wanting to know and experience mathematics in a way consistent with the vision of mathematics teaching and learning provided in my mathematics education courses.

The question of knowing and bothering.

How do these bothering questions impact my being as a learner and doer of mathematics? According to www.dictionary.com, the word, bother, means to take the time or trouble to do something. It can also mean to put oneself out. Other related bothering words include difficulty, nuisance, inconvenience, worry, anxiety, hassle, unsettle, perturb and fret. As I continue to bother, I take heed of Husserl's (1913/1970) words as he describes how to explore the world around us by a return "...to the things themselves" (p. 252). I take time to revisit my encounters with mathematics, my classmates, and the instructors. Then I ask questions and write text that helps me to better understand my experiences and the experiences of others as we learned side by side in a doctoral level mathematics class. I continue to make an effort until the anxiety and

difficulty of knowing has been addressed. Heidegger (1927/1996) advises that we should turn toward our anxiety and allow it to open up the possibility of an authentic moment.

The *mathemata*, the mathematical, is that “about” things which we really already know. Therefore we do not first get it out of things, but, in a certain way, we bring it already with us (Heidegger, 1962, p. 276).

At what point in my schooling did I engage with mathematics as something I might already know? When did I stop? When did the question of bothering emerge? Did I start to wonder, why bother, when I was no longer encouraged to seek for the mathematics from within? Questions about early mathematical consciousness are hard to answer. Reflections about recent mathematical experiences in high school and undergraduate studies suggest that I was not engaging with the mathematics as if I were bringing understanding or ideas to the table.

When we let children explore mathematical ideas on their own, uninterrupted by adult notions of doing mathematics, they naturally come to make sense of number and space in their everyday lives. Carpenter, Corbitt, Kepner, Lindquist, and Reys (1981) found that prior to formal schooling and mathematics instruction, young children are very successful in solving simple addition and subtraction word problems. However, it is common for older children to struggle when attempting to solve word problems (Zweng, 1979). This pushes me to ask why children who are early on in their mathematical development and thinking find word problems easy to solve while older children who have had more experiences, knowledge, and schooling struggle with the same problems? Why don't we let children continue to find the mathematics from inside? Why don't we continue to let them bother?

...little kids-preschoolers- as vibrant, alive learners, asking questions and seeking answers to their important questions. Preschoolers think. Moreover, they think about what's important to them. In this way, they learn. When those same vibrant, alive kids hit school, something happens. They begin to learn that their questions aren't the important ones. The important ones are the teacher's. And the major activity of school becomes finding answers to teachers' questions. How one gets the answers apparently doesn't matter because school seldom deals with that process. (Harmin & Gregory, 1974 p. 25)

For years, school mathematics has disconnected the ways young students naturally think about and solve problems in their daily lives. Schools have traditionally focused on procedural and symbolic aspects of solving problems. Problem solving was reserved to the word problems found at the conclusion of naked number exercises in classroom texts.

A Pedagogy that was Transformational

Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis? (Halmos, Paul R.)

I had stopped the fight so long ago that I can't remember the last time I asked my own questions- the authentic, deep, curious questions in mathematics class. I can't remember the last time I had my own examples or "discovered" anything worthwhile in mathematics class. In the spring of 2004, I enrolled in a mathematics class entitled, *Foundations of Mathematics: Research Experiences in Mathematics For Mathematics Educators (FM)*. It was in this class that I was led with all my senses. We traveled together to some of the most unfamiliar places. Our teachers quickly morphed from being teachers to becoming our mathematical tour guides. Familiar with the terrain, the culture, and the weather of mathematics, they were able to lead. It was in this class that I bothered to start the fight.

The class was like no other I had taken. The instructors and the mathematics welcomed intuition and curiosity. There was a beauty in its unstructured structure and a natural desire to travel with each other, and with the mathematics, on our learning journey. The class provided me an opportunity to value other ideas and notions in mathematics beyond the content. I was allowed and encouraged, eventually expected, to engage all my senses in exploring mathematics. It was in this class that I boldly raised my hand and with a confident voice, eager to understand asked, “WHY BOTHER?” Asking the question signified the end of a long silence in my understanding of mathematics and opened the possibility for a fruitful mathematical future filled with authentic questions.

Today I asked the, “Why bother?”, question. I was being honest. I did not ask the question out of disrespect for the class nor the mathematics. As I consider why I asked the question, perhaps it was what one of my third graders might ask, or a preservice teacher or parent. I’ve thought about this episode a lot. A popular athletic slogan comes to mind– For the love of the game. This would explain why I might practice free throws or a particular play over and over– for the love of the game. But, I know basketball and can see the point in bothering to practice– for the good of my game. Mathematics is different. I know some fundamentals but maybe I don’t know enough to automatically see why I should care about a relationship between means [arithmetic and geometric] (personal journal, 02.10.04).

The course.

The Foundations of Mathematics course was intended to provide mathematical experiences for mathematics education doctoral students. The course goals were to help students (a) develop strategies for continuing to learn mathematics throughout their lives, (b) to become a community of mathematics scholars and (c) to create an environment in which participants conduct serious mathematics investigations.

The course was unique in many ways. There was no textbook, nor were there quizzes or exams. The content of the course was driven by mathematical problems, mostly problems that evolved from the contribution of students in the class. The out of-

class project was an opportunity to spend a considerable amount of time learning how to pose our own mathematical questions and then set out to understand possible answers to them. The roles of teacher and student were unique in that the students voice was just as much a driving force as were the teachers'. A more detailed description of the course is provided in Chapter Four.

Definition of the Primary Research Problem

As mathematics education doctoral students, we have had the benefit of studying mathematics and learning about mathematics education. Collectively, we have spent years teaching, learning, and trying to connect our teaching with our learning. We have had the opportunity to study the history of mathematics education in our country and all of the mathematics education movements in curriculum and pedagogy. We have had the opportunity to engage in deep conversations with respected scholars in our field and to read about and conduct research in our field. We have had the opportunity to read about and make sense of the NCTM reform documents that have impacted the reform movement tremendously in mathematics education in the past 20 years. For those of us who have had classroom teaching experiences, we have been able to teach through these ideas and challenge our children to engage fully with the mathematics we teach. With these children, we have been able to enact the verbs of doing mathematics that the NCTM suggests: explore, justify, represent, discover, explain, predict, discuss, investigate, conjecture, formulate, and construct (NCTM, 1989).

As the time comes nearer to step into academia as mathematics educators, it is the knowledge, experiences, and opportunities that have been lacking in our preparation that has become bothersome. As previously stated, we will be expected to know and do a long

list of things. This study will examine an aspect of our doctoral experience that may be lacking in our preparedness to go out and be stewards in mathematics education. I posit we have not had sufficient opportunities to experience mathematics as learners in a way that we have read about, discussed, researched and taught to our own students.

As a learner and teacher, I now am able to come to this place where I am free to ask not only the bothering question, but also the other questions pushing out from inside:

1. What is the nature of the mathematical experiences that mathematics education doctoral students identify as components of their program?
 - What is the meaning of these experiences in the context of our doctoral student careers?
 - How are these experiences situated within in our preparedness as we enter the mathematics education field?
2. How can specific mathematical experiences influence the identity and preparation of a mathematics education doctoral student?

Section Two: Why do this Research?

According to Niss (1999), mathematics education research is, "the scientific and scholarly field of research and development which aims at *identifying, characterizing, and understanding* phenomena and processes actually or potentially involved in the *teaching and learning of mathematics at any educational level*" (p. 5). The purpose of this study is to understand the phenomena surrounding the learning of mathematics at the doctoral level in mathematics educational doctoral programs. I do this work not to prove or prescribe but rather to uncover, or provide a vision for stakeholders in mathematics

education doctoral programs and mathematics education writ large.

There are multiple stakeholders in mathematics education: K-12 schoolteachers, college and universities professors and instructors, elementary through college aged students, parents, government, and society. It seems a worthwhile endeavor to understand the perspective of each stakeholder, as well as to provide opportunities for different groups to hear the voices of others who have a vested interest in mathematics education.

NCTM and others organizational stakeholders have helped to provide a vision of model mathematics teaching and learning- this has been largely focused at K-12 classrooms. There is not as clear a vision for mathematics teaching and learning post-high school. This lack of vision as well as the tradition of mathematics at universities and the constraint of large class sizes perpetuates the factory like process of credit accumulation in many programs.

As future mathematics teacher educators, mathematics education doctoral students have a stake in mathematics education as well. Considering mathematics teacher education, Lapan and Rivette (2004) suggest:

We have reached a place in our profession where we recognize that the only defensible directional goal is to commit ourselves to successive approximations toward the nirvana where every teacher of mathematics has the right stuff and a fire in his or her belly to reach every child. Consequently, mathematics teacher educators need to build a profession that thrives on the challenge of examining, with curiosity, commitment, and vigor, the hard problems inherent in the teaching and learning of mathematics and the preparation and lifelong support of K-12 teachers.

One of the aims of this research seeks to capture the essence of learning mathematics in a “reform inspired” graduate level course in the context of the broader mathematical experiences of doctoral students in mathematics education. I wish to more

deeply understand my own experience as a student and the experiences of others in the class. I hope my research will reveal how the experience affected us generally, and more specifically how it might affect us as we step into our future roles. The course is testimony that this kind of mathematical encounter can work at all levels and is equally important for young students as well as future mathematics educators. Ideally, I hope this work will cause consideration of such experiences for all mathematics education doctoral students.

In the *Advancement of Learning: Building the Teaching Commons*, Huber and Hutchings (2005) suggest that students' voices must be a part of the discussion about learning in higher education and how to improve it as they gain and add powerful, new insights when they participate in such conversations (p.118). This study is unique in that it reveals the experiences of mathematics education doctoral students from their perspective and thus provides a voice for students in that conversation.

Who Else Might Care?

Eager to understand for myself, I am conscious of those in the field who might also find this piece of research worthwhile. I am hopeful that this research would provide insight into the following areas.

Mathematics education community.

Fueled by results published in *A Nation At Risk* (The National Commission on Excellence in Education, 1984) and international achievement tests such as the *Trends in International Mathematics and Science* (National Center for Education Statistics, 1998), new visions have emerged of improving the state of mathematics teaching and learning in the United States. The National Council of Teachers of Mathematics developed several

seminal documents in an effort to help promote change for our students. Although some progress has been made in improving mathematics instruction in this country, the journey is far from over. Many argue that improving mathematics teaching by strengthening the effectiveness of more of our mathematics teachers is one important challenge. In order to strengthen the effectiveness of our teachers, there is a need to consider the quality of teacher preparation, specifically in mathematics content and pedagogy. Perhaps paying attention to the kinds of experiences and learning exposed through this study would be fruitful in those discussions.

Mathematics education doctoral programs.

“As one leading scholar of doctoral education, Charlotte Kuh, frequently opines, “We measure what we value, and we value what we measure” (Nyquist, 2002). Thus, a key step toward realigning doctoral student expectations, experiences, and outcomes will come with systematic and comprehensive assessment, ranging from individual students to doctoral programs. Research efforts have begun to assess mathematics education doctoral programs. This study will add a key piece to this research: the mathematical experiences of mathematics education doctoral students.

Student experience.

A call for a greater democratization of the mathematics classroom through a shift in authority from the teacher to the students exists in mathematics education today. Research in the field focuses on a wide range of issues surrounding the teaching and learning of mathematics and has provided a rich theoretical field for the mathematics education research community (Lerman, 2001). If the mathematics education community is to understand the complexity of mathematics education issues in their entirety, I posit

that such democratization be applied to mathematics education research as well. This shift in research demands seriously considering the perspectives of students.

The voicing of student experience is rather neglected in most educational research (Erickson & Schultz, 1992). When student experience is cited, it is typically reported from the perspective of an adult educator or researcher, rather than directly from the student's mouth as a first-person account of the phenomena of students' experience. Of these few studies, many utilized a questionnaire or survey to collect information *about* student experience. The studies that provided the most promising insight into student experiences are those utilizing actual student writing, written work, and writing assignments (Erickson & Schultz, 1992). As noted by Erickson & Schultz (1992), John Dewey's dedication to student experience has caught on in some degree to curricular and methodological issues, but has yet to be embraced by those engaging in educational research.

In this chapter, I introduced the study and provided an overview of my mathematical experiences and preparation that have influenced my coming to this study. Next, I described the importance of this study and included a brief description of how this study fits into the broader arena of mathematics education. In the following chapter, I lay out the theoretical framework and present relevant literature for this study.

CHAPTER TWO

This chapter consists of two sections. The first section outlines the theoretical framework for the study. The veins of the framework describe ways of learning and knowing; learning and knowing in mathematics education; ways of experiencing and doing mathematics; and being and becoming. In addition, this chapter contains a review of the literature that has informed this study. This includes but is not limited to literature pertaining to nature of mathematics teaching and learning, mathematics education reform, and doctoral student preparation.

Theoretical Framework

The most complicated matter on this dissertation journey has been the declaration of a theoretical perspective. My theoretical perspective affects the way in which I come to understand my experiences and those of others. It also influences the authors I choose to read and the literature I choose to draw upon. As a neophyte researcher, I am hesitant to declare permanent residency in a particular theoretical camp. However, it is the following ways of knowing that I find myself returning to. Collectively, they seem to best describe my “theoretical-ness”. I find comfort in knowing that like my “knowing” of mathematics, I expect change and challenge of my perspective. I consider the following a map that highlights majors ports of call for my thinking and communicates to others where I am drawing the meaning of my journey.

I start with a fundamental belief that the humanness in the pursuit for, acquisition of, and practice of the doctorate degree should take precedence over the discipline and advancing knowledge within the discipline. Without the students, faculty, knowers, and doers of the discipline, the knowledge of the field is fruitless. If one chooses to view

doctoral education from that perspective, Golde & Walker (2006) suggest that we ought to put students front and center when examining the adequacy of the programs in *Envisioning the Future of Doctoral Education: Preparing Stewards of the Discipline - Carnegie Essays on the Doctorate*. “Thus those responsible for creating, evaluating, or modifying doctoral programs should directly and explicitly broaden their focus by thinking about the ways in which departing students go out into the world to develop and transform their discipline by applying the knowledge, skills, and experiences they acquire as doctoral students” (p. 47).

I posit that mathematics education doctoral students experiences are shaped by three major components: the experiences, knowledge, and beliefs that we bring with us to the doctoral program, the experiences and interactions in our mathematics education doctoral program, and the future responsibilities that our field expects-that which we are to be. If I am to take these as givens, I look to the following theoretical ways of learning, knowing, being and becoming.

Ways of Learning and Knowing

From the late nineteenth century to the present, two strikingly different visions of teaching and learning have been competing for primacy in American schools. They have been by a variety of names...the most common labels, which capture most of the sense of these various category systems, are teacher-centered vs. child-centered (or student-centered), traditional vs. progressive, and, in what is currently the most popular terminology in education schools, traditional vs. constructivist teaching (Labaree, 2004, p.130).

This study draws on the belief that learning is not defined as the acquisition of information or knowledge. Rather, learning is in the relationships between people making it a genuinely situated and personal experience. In most instances, this view of learning

contradicts the learning that I experienced for most of my own academic career. My philosophy rests on a belief that teachers, regardless of the age of their students, must actualize the humanness in teaching by establishing personal relationships with students and fostering safe learning environments.

To understand differences between the approaches to teaching, learning, and schooling, I look to educational philosopher John Dewey's *The Child and the Curriculum*:

*One school fixes its attention upon the importance of the subject-matter of the curriculum as compared with the contents of the child's own experience...
...Subject matter furnished the end, and it determines method. The child is simply the immature being who is to be matured; he is the superficial being who is to be deepened; his narrow experience which is to be widened. It is his to receive, to accept. His part is fulfilled when he is ductile and docile.*

Not so, says the other sect. The child is the starting-point, the center, and the end. His development, his growth, is the ideal. It alone furnishes the standard. To the growth of the child all studies are subservient; they are the instruments valued as they serve the needs of growth. Personality, character, is more than subject-matter. Not knowledge or information, but self-realization, is the goal...Moreover, subject-matter never can be got into the child from without. Learning is active. It involves reaching into the mind. It involves organic assimilation starting from within (p. 12-13).

In a constructivist framework, experience, interaction and reflection are all key sources of knowledge for conceptual advancement. It is important to the learner's experience to talk about their thoughts, talk to each other and talk with the teacher. "To verbalize what one is doing ensures that one is examining it. And it is precisely during such examination of mental operating that insufficiencies, contradictions, or irrelevances are likely to be spotted." (vonGlasserfield, 1991, p. xviii). Learning and knowing are the result of the student's activity. Learning is not about the passive reception of information (Cobb, 1988, Fostnot, 1996; von Glasserfield, 1990).

Learning and Knowing in Mathematics Education

There are two ways to teach mathematics. One is to take real pains towards creating understanding- visual aids, that sort of thing. The other is the old British system of teaching until you're blue in the face. (Newman, 1956)

Traditional.

In the traditional method of mathematics teaching, several specific theories and models have emerged including behaviorism, direct instruction, back to basics and recitation. All of these share similar characteristics and can be seen regularly in mathematics classrooms today. In general, these models focus on a decrease in importance of thinking and reasoning and an increase in students' acquisition of facts, skills, and procedures. Dewey suggests that these traditional approaches rely on "bodies of information and of skill that have been worked out in the past; therefore, the chief business of the school is to transmit them to the new generation" (1938, p. 17).

Most traditional models take a curriculum or content and break up the information into small chunks of knowledge. These pieces, or skills, are to be mastered in a predetermined order. According to these models, the learner will learn these skills by listening to teacher explanations, observing, engaging in experiences, and practicing the skills. Each skill should be learned to a mastery level. Then the skills and knowledge can be put together by the student to understand or produce the whole. Learners are seen as passive bodies in the learning process, almost absorbing much of the information. Learning is considered a system of behavioral responses to physical stimuli. There is an assumption that the learner will be able to put together the skills and knowledge to understand or produce the whole. The line of discourse is generally one-way. The teacher talks, asks questions, and the students respond. Students rarely speak to each

other about the mathematics. In this model, there is an assumption that students will absorb and process the incoming information and be able to generalize it and make sense of it on their own.

In many classrooms, this means drill, repetition, and memorization. One of the foundational beliefs of traditional mathematics teaching is the notion that mastery of certain facts and skills is a prerequisite for all other learning. For example, a teacher may not provide any further material until the students in her room have all memorized their multiplication facts from 1-12. The assumption that students can't move on to higher level mathematical ideas can be limiting to the kinds of learning that occur in the classroom as well as what occurs as several years go by.

Constructivist.

The National Council of Teachers of Mathematics has established a vision of high quality mathematics teaching and learning for all students. For this vision to become a reality, the NCTM suggests students take on a role that is analogous to the role of a mathematician and includes students engaging in the exploration of mathematics, creating mathematics, and evaluating mathematics (NCTM, 1991; 2000).

The NCTM standards- based models and other reform theories have evolved from the work of Jean Piaget, Lev Vygotsky, and John Dewey among others. Their work provides the foundational tenants of constructivism, which influences many of today's reform curricula. Constructivism considers the developmental stage of the learner and focuses on students' understanding of conceptual knowledge, unlike traditional models that encourage students to become fluent in procedures while minimizing the importance of concepts.

The constructivist theory of teaching is clearly a response to the constructivist assumption of how students learn. In practice, constructivist beliefs would be reflected in teachers' actions, behaviors, instruction, content, pedagogy, and teachers' and students' roles in the classroom. This perspective suggests, according to Cobb et al (1991 in Koehler & Grouws, 1992), that mathematical learning is not about learning individual bits of knowledge as it is delivered in a classroom but is instead a process of reorganizing activity. The activity is interpreted to include conceptual activity or thoughts.

Social Constructivism acknowledges that all human knowledge can be constructed through interactions with each other as well as by their own personal processes. Meaningful dialogue and discourse are vehicles for social constructivism. Language shapes and is the product of individual learning. Social constructivist theory emphasizes students communicating mathematically and problem solving with each other. Constructivist theory of mathematics offers students a chance to explore, discuss, and challenge mathematical ideas with other students. These related theories rest on the assumption that these opportunities to explore, discuss, and challenge ideas will enrich the thinking of students and help them to make connections with their thoughts and the thoughts of others (Merrill, 1991).

Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding. To understand what they learn, they must, enact for themselves verbs that permeate the mathematics curriculum: “examine”, “represent”, “transform”, “solve”, “apply”, “prove”, “communicate”. This happens most readily when students work in groups, engage in discussions, make presentations, and in other ways take charge of their own learning (National Research Council From *Everybody Counts* 1989, pp. 58-59).

Ways of Experiencing and Doing Mathematics

We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work. (Feynman, Nobel Lecture, 1966).

Mathematicians, the research mathematics community, and mathematically literate adults practice mathematics very differently from the way mathematics is practiced in schools (Richards, 1991). I draw upon Richard's framework of the various ways that mathematics is known and is done in particular communities. In this framework, there are four communities, each with different assumptions, goals, and underlying methodologies for what it means to know and do mathematics. These communities, or cultures, are: research math, inquiry math, journal math, and school math. Richards describes their differences in detail (p. 15):

Research Math- the spoken mathematics of the professional mathematician and scientist. Professional mathematicians share much of their discourse structure with other research communities in that the language is technical and makes many assumptions regarding the underlying content area...The mathematical research community discourse is structured according to a "logic of discovery" (cf. Popper [1959]) stressing the actions of making conjectures and refutations. The characteristic that distinguishes mathematicians from other research communities is their subtle reliance on notions regarding the nature of proof.

Inquiry Math- mathematics as it is used by mathematically literate adults. This approximates, but is still distinct from the language of the "mathematical research community." The language of mathematical literacy includes participating in a mathematical discussion, and acting mathematically – asking mathematical questions; solving mathematical problems that are new to you; proposing conjectures; listening to mathematical arguments; and, reading and challenging popular articles containing mathematical content.

Journal Math- the language of mathematical publications and papers. The emphasis is on formal communication, at a distance or across time, where there is no opportunity to clarify ambiguities. This language is very different from the spoken language of the research community, that is, it is very different from a logic of discovery. Papers and publications are based on a "reconstructed logic" that makes

mathematical discoveries more palatable for public consumption (cf. Richards [1988])

School Math- the discourse of the standard classroom in which mathematics is taught. This discourse does not differ much from “school science” or “school English”. Viewed as discourse structures, classroom lessons mostly consist of “initiation-reply-evaluation” sequences (cf. Mehan [1979]). What is learned is useful for solving habitual, unreflective, arithmetic problems. In many ways, this discourse does not produce mathematics, or mathematical discussions, but rather a type of “number talk” that is driven by computation.

Richards identifies the similarities between *research math* and *inquiry math*. Both communities of mathematics are based on a “logic of discovery” In contrast, *school math* and *journal math* are similar in that they are dependent on a “reconstructed logic” and are designed to “transfer information to a community that has already accepted many presuppositions and constructed a great deal of mathematics. Richards contends that this is, “useless for students who are faced with the need to construct these presuppositions and to construct mathematics for themselves (1991, p. 16). He notes that the school mathematics focus on the product of mathematical activity hinders students from having opportunities to do mathematics for themselves which he sees as personally meaningful and an important step in learning how to engage in mathematical discussions as mathematically literate individuals. “Obviously, “ says Richards “*school math* is the wrong math to be teaching students” (1991, p.16).

On Being and Becoming

A clash in preparation and preparedness has been increasingly intensified in the past few decades. The clash reflects the differences in student expectations and experiences and program philosophy and requirements. The Ph.D. is primarily a research degree. However, not all Ph.D. students aspire to faculty careers in research. In addition, not all Ph.D. programs aspire to prepare students for faculty careers or for the full range

of colleges and universities. In fact only about 1 out of 10 new Ph.D.'s will end up working at a research university (Gaff & Lambert, 1996) There is increasing dissatisfaction with the job readiness of new Ph.D.'s. The Carnegie Foundation of the Advancement of Teaching reports that many new Ph.D. recipients are ill-prepared to function effectively in the setting in which they work and are especially plagued with roles related to teaching.

Hyman Bass, mathematician and scholar, suggests that doctoral programs ought to be training "T-shaped" individuals. That is, by the time students receive their Ph.D.'s, they would have sufficient depth of knowledge in a narrow area, represented by the vertical stroke of the "T" as well as significant breadth of knowledge of the field represented by the horizontal stroke of the "T" (cf. Taylor, 2006, p. 51). President of the Carnegie Foundation for the Advancement of Teaching Lee Shulman describes the doctorate as, "a degree that exists at the junction of the intellectual and moral (cf. Golde, 2006, p.3). He continues, "The Ph.D. is expected to serve as a steward of her discipline or profession, dedicated to the integrity of its work in the generation, critique, transformation, transmission, and use of its knowledge". Both scholars agree on the seriousness and complexity of the nature of preparing doctoral students in mathematics education.

Stewards of the Discipline.

The Carnegie Foundation for the Advancement of Teaching established the Carnegie Initiative on the Doctorate (CID). The CID is a five-year research project focused on "aligning the purpose and practice of doctoral education" (Golde, 2006, p.6). With a framing question posed, "If you could start de novo, what would be the best way to

structure doctoral education in your field?”. Academics across several disciplines were charged with responding via writing essays. Three additional questions were raised:

1. What constitutes knowledge and understanding in the discipline?
2. What is the nature of stewardship of the discipline?
3. How ought Ph.D.’s be educated and prepared? (Golde, 2006, p. 9).

The *Merriam-Webster* online dictionary (www.m-w.com) defines stewardship as “the careful and responsible management of something entrusted to one’s care (cf Golde, 2006, p. 12)”. Specifically, “Stewards think into the future and act on behalf of those yet to come. A Steward of the Discipline, then, thinks about the continuing health of the discipline and how to preserve the best of the past for those who will follow. Stewards are concerned with how to foster renewal and creativity. Perhaps most important, a steward considers how to prepare and initiate the next generations of stewards (p. 13).”

The Carnegie Foundation provides a model for the charge that many universities have of preparing doctoral students for becoming “Stewards of the Discipline” (Carnegie Foundation, 2005). This model describes doctoral students as a process in which doctoral students become stewards of their disciplines by being purposeful about (a) the creation of new knowledge, (b) conservation of existing knowledge, and (c) transformation of knowledge to others.

A particular complexity in mathematics education doctoral programs complicates the charge of becoming “stewards of the field”. Our identity in mathematics education is deeply rooted in traditions of education and mathematics- stewards who shall be responsible for the well being of the learner as well as the well being of the mathematics. According to the National Research Council (1992), doctoral study in mathematics is

mainly focused on traditional, theoretical mathematics. And so, as a result, “many researchers lack the broad knowledge needed to address real world problems...and the system of education is more or less self-contained, with graduates teaching what they have been taught in the same manner they have been taught (p. 15).” Doctoral programs in mathematics education are very often housed in colleges of education (Reys & Kilpatrick, 2001) . The discipline for which we are responsible in sharing with others is typically housed in a college or department different from the one in which we reside. Each house, the education and the mathematics, have separate histories, knowledge and philosophy of what stewardship may mean. Mathematics departments, traditionally, are department where the purpose and interest is in the mathematics, not explicitly the transmission of that knowledge.

Literature Review

Mathematics education doctoral students come to doctoral work with past experiences, knowledge, and beliefs that influence how each will experience the doctoral program. We enter mathematics education doctoral programs with our mathematics educator/research identities partially constructed. We are all former K-12 students. Some of us have been K-12 teachers, likely following a traditional preservice training program. Therefore I look to literature on the previous research on mathematics education doctoral studies, improvement of mathematics doctoral studies, related teacher preparation, NCTM recommendations, and student experience in mathematics to help situate my work.

Mathematics Education Doctoral Programs

Information about the nature of mathematics education doctoral programs in the United States is slim and not comprehensive. Despite the fact that some surveys have been conducted on U.S. doctoral programs in mathematics education, current and substantive information is not available (Reys & Kilpatrick, 2000, p. xi).

The early twentieth century witnessed the first doctorates in mathematics education in the United States (Reys & Kilpatrick, 2000, p. xi). The field has grown in the past several decades. According to Reys (2000), the number of doctorate degrees awarded in mathematics education has ranged from 50 in 1982 to 115 in 1998, with a yearly average of about 70.

According to the National Research Council (NRC), 1,386 doctoral degrees with a major field of mathematics education were earned in the United States between the years 1980 and 1998 (Reys, 2000, p. 1267). The nature and focus of these doctoral programs varies greatly as they were awarded by 126 different institutions each with unique objectives, requirements, and faculty. For example, one university's mathematics education doctoral program may focus heavily on mathematics content with some select courses in education while another university's program may focus heavily on mathematics education with some select mathematics courses. In addition to the differences in specific programs, there are institutions lacking in established mathematics education doctoral programs that are preparing doctoral students to become mathematics educators. The diversity of the programs creates diversity in the preparation and professional characteristics of the mathematics education doctorates (Reys, 2000, p. 1267).

The diversity of preparation of people holding doctorates in mathematics education is confronted with the increasing expansion of job opportunities in mathematics education (Reys, 2000, p. 1267). Traditional positions in departments of education are being supplemented by job opportunities in university mathematics departments, schools/departments of education, school districts, governmental agencies, publishing houses, and testing companies (Reys, 2000, p. 1267).

Doctoral education has been a subject of criticism for decades. According to Golde and Dore (2001), a great deal of attention was paid to doctoral education in the 1990's via reports and studies (Bowen & Rudenstine, 1992; Committee on Science, Engineering, and Public Policy, 1995, Association of American Universities (1998); & the National Science Board (1997). The attention, Golde and Dore posit, was largely spurred by the changes in the academic job market. A need to re-envision doctoral education has been made clear by recent studies and reports (Connolly, 2002). These most recent reports suggest a shift in preparation for a career in traditional academic research since approximately 1 out of 10 new Ph.D.'s will work at a research university (Gaff & Lambert, 1996). This recent rush of studies is helping us to better understand doctoral student experiences but absent from this data is still the "actual experiences of doctoral students" (Golde & Dore, 2001, p. 20).

According to a survey study by Golde and Dore (2001), many contemporary doctoral students "experience a mismatch between their doctoral training and their subsequent career opportunities. Mismatches occur on a variety of levels. A mismatch could exist between that which happens in the program (what you do and learn) with what you do after the program. For example, there appears to be a mismatch in the heavy

focus on research in coursework and dissertation while the actual careers of most PhD does not call on these skills and dispositions on a regular basis. The report, *At Cross Purposes: What the Experiences of Doctoral Students Reveal About Doctoral Education*, released in 2001, was a national survey of over 4,000 doctoral students from 27 universities and one cross-institutional program representing 11 arts and sciences disciplines. Their survey was based on the assumption that “students’ experiences reveal how the system is functioning- what is working and what is not” (Golde & Dore, 2001, p. 20).

Focus on Improvement of Mathematics Education Doctoral Programs

A National Conference on Doctoral Programs in Mathematics Education was held in October, 1999, at Lake Ozark, Missouri. The purpose of the conference was to begin the process of examining, discussing, and making recommendations for doctoral programs in mathematics education. The conference, funded by a grant from the National Science Foundation, was preceded by a year of preparation that included a comprehensive survey of programs. The conference was designed to provide a dialogue regarding

- the nature of current doctoral programs in mathematics education,
- ways of strengthening such programs, and
- suggestions and guidelines for faculty engaged in restructuring an existing programs or creating a new one (Reys & Kilpatrick, 2000p. xii).

Two pivotal questions surfaced during the conference: Is there a “core or canon of knowledge” needed by those earning doctorates in mathematics education? And, What are the essential elements of a doctoral program in mathematics education? (Association of Mathematics Teacher Educators (AMTE), 2002).

A task force was charged with creating a set of principles intended to “guide and design doctoral programs in mathematics education. The task force identified specific “core knowledge” expectations that are believed to be essential for graduates of doctoral programs in mathematics education. The expectations speak to a variety of topics including mathematics content, research, educational contexts, learning, teaching and teacher education, and technology. With respect to mathematics content, the task force states the following:

Mathematics educators need broad and deep mathematical knowledge both to identify the big ideas in the pre-K–14 mathematics curriculum and to examine how those ideas develop throughout the curriculum. Regardless of the entering level of mathematical knowledge they bring to a doctoral program, students should continue to study mathematics while in the program. Although each student may follow a different program of study, all should exit the program with some graduate study of mathematics and a deep and broad understanding of pre-K–14 mathematics. Standard courses in advanced mathematics are appropriate for students pursuing some goals, but such courses are seldom consciously designed or delivered in ways that enhance the knowledge or understanding of pre-K–14 mathematics. Avenues to accomplish broad understanding could include combinations of the following: formal mathematics course work, special courses or seminars examining specialized (pre-K–14) mathematics from advanced points of view, and clinical experiences in curricular development with intense scrutiny of the interconnectedness of different mathematical strands (AMTE, 2002).

Preservice Mathematics Teacher Education

Calls for reform in mathematics education promote a new vision of school mathematics (NCTM, 1989, 1991, 1995, 2000; National Research Council [NRC], 1989, 1990). This vision gives a picture of classrooms where teachers and students are *doing* mathematics. The NCTM defines the *doing* of mathematics to be an activity that involves but is not limited to exploring, solving, justifying, discovering, verifying, investigating, formulating, predicting, and explaining (1991). The classrooms in which students and

teachers are *doing* mathematics are supportive of mathematical inquiry and sense-making. This is a shift from focus on learning and practicing routine skills and drill.

In order to actualize these visions, mathematics teaching and learning must reexamine all levels. One of the most important is examining the preparation of mathematics teachers. Who is preparing these teachers? In most cases of traditional routes to professional certification in education, it is mathematics educators in schools and colleges of education that are teaching mathematics methods courses. Future teachers are also typically required to take mathematics courses, most of which are offered in the mathematics departments. With the vision in mind, teacher education departments are now being called to teach prospective teacher to learn to teach in ways that promote mathematical inquiry and conceptual understanding. Most teachers have not experienced learning mathematics in this way themselves. This creates an added complexity in the learning to teach as most teachers hold on to the ways in which they themselves learned.

Preservice teacher education, with respect to mathematics education, has been the focus of much recent research and debate. Questions about what knowledge is important for prospective elementary mathematics teachers to possess and what features of an teacher preparation program encourage prospective teachers to look critically at their knowledge and beliefs about what it means to know, learn, and teach mathematics are at the center of these discussion. It is widely accepted that teachers in existing teacher preparation programs may not possess the important mathematical knowledge basis for prospective elementary mathematics teachers. A reason for the lack of mathematical

sophistication lies in the nature and quantity of college level mathematics courses that prospective teachers are required to take.

Content Knowledge.

Ma (1999) found that limited subject matter knowledge restricts a teacher's capacity to promote conceptual learning among students. Ma's research showed that the way in which a teacher tended to help students depended heavily on his/her own knowledge of the topic. As educators prepare students for more challenging mathematics content, the need for prospective teachers to learn more challenging mathematics and how to teach it becomes vital.

Because of the depth and breadth of elementary level mathematics, improving teacher mathematical knowledge should be a key component in a teacher preparation program (Conferences Board of the Mathematical Science [CBMS], 2001; Wilson, Floden, & Ferrini-Mundy, 2001). This includes ensuring that prospective teachers become mathematically competent. Teachers must enter their classrooms believing that mathematics is centered on sense making. They must also be able to follow students' mathematical thinking and understand the reasoning behind students' errors and misconceptions (CBMS, 2001).

The following components, suggested by the Conference Board of the Mathematical Science (2001), regarding the topic of improving the content knowledge of teachers are critical in designing an elementary mathematics methods course. These suggestions include the following:

- Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.

- Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical "common sense" in analyzing conceptual relationships and in solving problems.
- Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching (CBMS, 2001).

National Council of Teachers of Mathematics

The National Council of Teachers of Mathematics, born in 1920, has become the world's largest mathematics education organization. NCTM's collection of publications and resources includes five professional journals, extensive on-line literacy, and over 200 books. Of these, several stand out as key literature in guiding reform in mathematics education: *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), *Professional Standards for Teaching* (NCTM, 1991), *Assessment Standards for School Mathematics* (NCTM, 1995) and most recently the *Principles and Standards for School Mathematics* (NCTM, 2000). These documents guide reform in mathematics education by providing the research and framework for establishing a vision of high quality mathematics teaching and learning.

We challenge all who have responsibility for any part of the support and development of mathematics teachers and teaching to use these standards as a basis for discussion and for making needed change so that we can reach our goal of a quality mathematics education for every child (NCTM, 1991, p. vii).

It was the development of each of these documents that would be the impetus for positive and sustained change in mathematics education as laid out. Specific

recommendations for the professional development of teachers were made. “The kind of teaching envisioned in these standards is significantly different from what many teachers themselves have experienced as students in mathematics classes (NCTM, 1991, p. 2).”

The *Professional Standards for Teaching Mathematics* provides guidance for five distinct components of teaching and instruction (p. 5):

1. Standards for teaching mathematics
2. Standards for the evaluation of the teaching of mathematics
3. Standards for the professional development of teachers of mathematics
4. Standards for the support and development of mathematics teachers and teaching
5. Next steps

The components most relevant to the nature of this study are those related to the professional development of teachers of mathematics and the support and development of mathematics teachers and teaching.

Standards for the professional development of teachers of mathematics.

The standards for the professional development of teachers of mathematics describes a vision for preparing teachers- not exclusively neophyte teachers but also experienced teachers at all stages of their careers. The standards, “focus on what a teacher needs to know about mathematics, mathematics education, and pedagogy to be able to carry out the vision of teaching” as described by the NCTM. The following aspects are addressed (NCTM, 1991, p. 6):

- Modeling good mathematics teaching
- Knowing mathematics and school mathematics
- Knowing students as learners of mathematics

- Knowing mathematics pedagogy
- Developing as a teacher of mathematics
- Teachers' roles in professional development

Six standards for the development of teachers of mathematics are given:

1. Experiencing Good Mathematics Teaching
2. Knowing Mathematics and School Mathematics
3. Knowing Students as Learners of Mathematics
4. Knowing Mathematical Pedagogy
5. Developing as a Teacher of Mathematics
6. The Teacher's Role in Professional Development (NCTM, 1991, p. 123).

The standards explicitly state that they are intended for the professional development of teachers of mathematics at the K-12 levels. The programs apply to introductory programs that prepare teachers but according to the authors, also apply to programs of advanced study for teachers of mathematics and other learning experiences in which teachers of mathematics may participate throughout their careers. The founding principles of this standard are based on the following assumptions (p.124).

1. The *Curriculum and Evaluation Standards for School Mathematics* provides the vision of mathematics education at the K-12 level that is the basis for the professional development standards.
2. Teachers are influenced by the teaching they see and experience.
3. Learning to teach is a process of integration
4. The education of teachers of mathematics is an ongoing process.
5. There are level-specific needs for the education of teachers of mathematics.

Standards for the support and development of mathematics teachers and teaching.

The *Professional Standards for Teaching Mathematics* are intended to provide guidance to all who are a part of the preparation of teachers including colleges and universities. The standards for the support and development of mathematics teachers and teaching speak specifically to those who are responsible for making decisions that affect teaching mathematics such as policy makers, schools, universities, and professional organizations. They are based on the notion that being a teacher is being a part of a learning community that continually fosters growth in knowledge, stature and responsibility.

To create teaching environments that encourage mathematical problem solving, communicating, reasoning, and connecting ideas- in short, mathematical inquiry and decision making- teachers must have access to educational opportunities over their entire professional lives that focus on developing a deep knowledge of subject matter, pedagogy, and students... Colleges and universities have major responsibility for not only the preservice and continuing education of teachers, but also for relevant programs that are responsible to today's and tomorrow's educational needs. (NCTM, 1991, p. 177-184)

Student Experience in Mathematics: Teachers as Learners

Mathematics education doctoral students embody the experiences and characteristics of many characters. We are learners. This is our primary role as we undertake the doctoral degree. However, we have been and continue to be teachers in a variety of settings: in elementary, secondary or post- secondary settings. Understanding the students' experience of mathematics has several implications for the field because experience is a strong influence on teachers' actions and beliefs. According to Ball, Lubiencki, and Mewborn (2001), by the time teachers begin their professional education careers, they have already received more than two thousand hours in "specialized

‘apprenticeship of observation’ which has not only instilled traditional images of teaching and learning but has also shaped their understanding of mathematics” (p. 437).

Borasi and Fonzi (1998) describe several reasons why teachers experiencing mathematics as learners has crucial implications for learning. Of course, the prime objective is that doctoral students learn more mathematics. Because research has shown that teacher’s beliefs about mathematics and teaching mathematics are formed mostly as a result of having been students in traditional mathematics classrooms (Thompson, 1992.) Current reform efforts are radically different from the traditional mathematics education that shaped many teacher’ beliefs about mathematics teaching and learning. Many teacher educators argue that teachers must have personal experience of alternate pedagogical approaches to shake these up.

Collins, Brown and Newman (1989) identified a three phase model in the process of how people learning a complex task:

1. Modeling – The learner observes and examines how an expert engages in the task.
2. Scaffolded practice – The learner engages in the task himself/herself, but with the help of an expert and/or of other supporting structures.
3. Independent practice – The learner engages in the task without support.

Borasi and Fonzi (1998) suggest that when the complex task is learning a novel approach to teaching mathematics, “experiences as learners” activities offer an especially effective vehicle for such modeling. According to Borasi and Fonzi, the situation plays out in the following way. First, teachers observe an expert mathematics teacher educator teach mathematics in a non-traditional way. Second, because teachers participate in this

instructional experience as learners themselves, they are in a unique position to examine how their students may feel about the new approach. As a result, they are in a better position to evaluate its potential advantages and drawbacks.

Research confirms the benefits of teachers experiencing mathematics as learners. This type of professional development experience is paramount in several professional development programs with documented success (Simon & Schifter, 1991; Schifter & Fosnot, 1993; Borasi, Fonzi, Smith & Rose, 1999). A systematic study conducted by Simon and Schifter (1991) in the context of one of these programs has specifically shown changes in teachers' beliefs and practices toward a more constructivist approach to teaching mathematics. Although mathematical "experience-as learners" were just a portion of the professional development, case studies and anecdotal evidence (Schifter & Fosnot, 1993; Borasi, Fonzi, Smith, & Rose, 1999) further confirm that "experiences-as-learners" were a critical element.

Borasi and Fonzi suggest that in order for the mathematical learning experiences to be effective, the experiences need to accomplish the following:

- Challenge the participants intellectually, regardless of their mathematical backgrounds or the grade levels they teach. Only under these conditions can teachers be genuine learners and benefit fully from participating in these instructional experiences.
- Be mathematically sound and address key concepts. In order to strengthen teachers' knowledge of mathematics and invite them to rethink the goal of school mathematics, these experiences must offer opportunities to learn worthwhile and significant mathematics.
- Allow for mathematical reflection and discussion in addition to mathematical

problem-solving. Doing so is essential to ensure that teachers revise and enhance their current understanding of key mathematical concepts and procedures, and do not just engage in “activities for activity sake.”

- Model non-traditional ways of learning and/or teaching mathematics. Participants must experience alternatives to traditional school mathematics in order to appreciate their potential for student learning.

In addition to these characteristics, reflective activities should embody the following:

- Reflective activities should occur after the learning experience is over, not during it. In this way, participants may find it easier to abandon their teacher roles as they engage in the mathematical learning experience and be genuine learners in it.

- There should be opportunities for individual reflections as well as group discussion. Participants need to make personal sense of the experience as well as hear other people’s insights and perspectives.

Conclusion

This chapter has described the theoretical framework for this study. In addition, I have presented a review of relevant literature. The literature and research presented illustrate the need for improvement in the preparation of mathematics teacher educators. Following this chapter is the first of three Mathematical Interludes. The next chapter reports on the methods and procedures followed during data collection and analysis of this study which aims at providing insights towards such improvements.

MATHEMATICAL INTERLUDE ONE

Jessica's Brownie

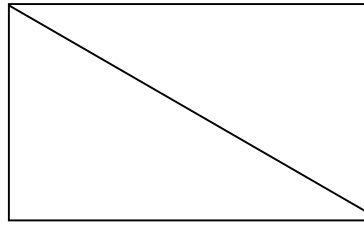
This interlude includes a reflection of a classroom episode that occurred while I was teaching third grade mathematics. Several years later, while enrolled in the Foundations of Mathematics course, I was required to choose a mathematics problem or question that I was personally interested in. I remembered the classroom interlude that had occurred. Returning to that episode, I continued to unpack the mathematics that was in question which eventually led me to a new geometric exploration. This work became the material of my “course project” during the first semester of my enrollment of FM.

Classroom Episode

Our third grade class had just started investigating the idea of “fair shares” as an introduction to understanding fractions. The activity we were engaged in was taken from the book “Fair Shares” from the *Investigations in Number, Data, and Space Curriculum*. For this activity, students were asked to cut out paper brownie rectangles and then find a way to share the brownie equally between two people. Student responses included the following strategies to share the brownie:

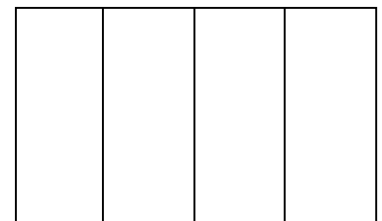
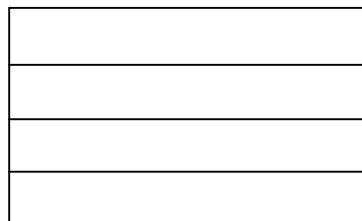
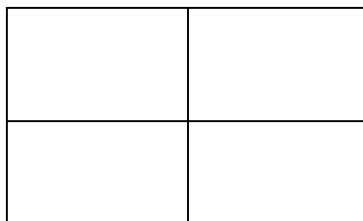


Except for Jessica who shared her brownie by dividing it like this:



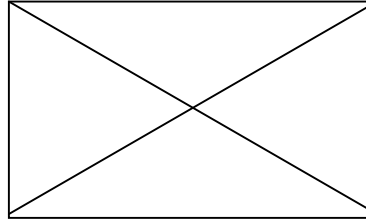
I had anticipated that students would make their cut either horizontally or vertically and hoped some students might make the diagonal cut. I was happy that Jessica had discovered this strategy. I asked students to share their strategies and provide some justification for how they knew their brownie shares in fact were fair. All of the students including Jessica were able to provide justification for how they knew their shares were equal. Most of them “proved” their equality by cutting up the brownie pieces and laying them on top of each other.

Then, I challenged students to take a new paper brownie and find a way to divide the brownie equally among four people. Student responses for four equal shares included the following strategies:



I noticed that Jessica hadn’t raised her hand to identify that she had chosen one of the three strategies that were shared. I asked the class if anyone had used a different

strategy even though I couldn't imagine how else any of my third graders would have made fourths. Jessica was eager to share her way:

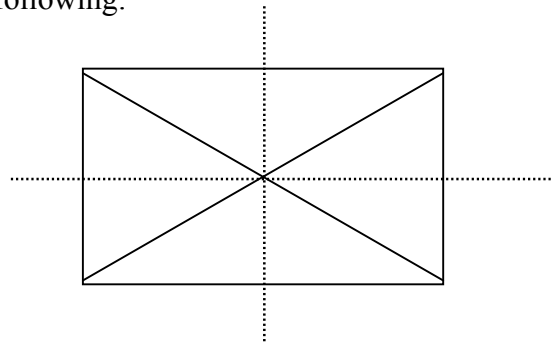


The students who used one of the more common strategies for making fourths were able to tell me how they knew that each share would be equal. They did this either by the cutting and comparing method or by a verbal justification about the result of equally dividing equal pieces. When it was Jessica's turn to explain her strategy, she was at first unable to prove if the four pieces were fair or not. The students in the room provided a resounding "NO" when I asked for their opinions on the fairness of Jessica's brownie pieces.

This particular episode became one of my most memorable "teachable moments". It is perhaps most memorable because I did not know the correct answer myself. It certainly felt like the pieces were equal; especially thinking about the equally-dividing-equal-pieces theorem that my students proposed. However, at that very moment I was challenged as to what to say and do next. We had a short discussion about why students felt these pieces could or could not be equal. We ended the lesson with uncertainty – but also with a challenge. Students were asked to go home and investigate the problem further and to try to find a convincing way to tell the class if the pieces were equal or not. That night, I thought about the problem as well and wondered what students would come up with. I couldn't remember any particular geometry proof about equally sharing pieces

such as these. How could I explain triangle congruence and similarity to third graders? What would that mean?

The next day, several students came ready to prove that the pieces were not equal- the pieces did not look like they were the same size and the students cut the four pieces out and laid them on top of each other to “prove” their point. Jessica still argued for fair shares and shared the following:



She explained that she divided each of the four triangles into two “little” triangles and found that the little triangles were exactly the same size and shape. She then explained that each of her fair shares was made up of two of those little triangles and that they must be fair even though they were made by different arrangements of the little triangles. Her explanation caused subsequent rich conversations for our class and filled me with questions about my mathematics teaching and pedagogy. What justification is appropriate and sufficient for a third grader? For a 10th grader? A third grade teacher? A geometry teacher? This problem hibernated within me until recently.

Revisiting the Problem

Six years after that class exploration, I am in my final year of coursework in graduate school. In a mathematics education course, I was asked to consider a mathematics problem that has been of interest to me. In particular, we were encouraged

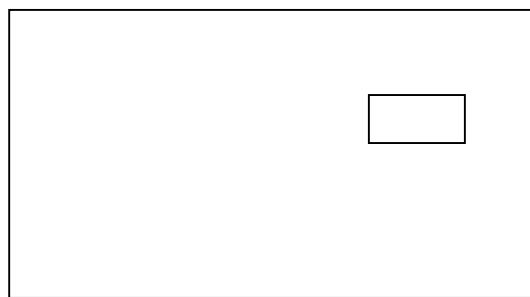
to revisit a topic that we had previously been unable to take the time to explore. I knew immediately that I was going to revisit Jessica's brownie problem.

I saw the strength in Jessica's argument. However, were there other ways to prove her problem? My exploration began with trying to officially prove the pieces were equal. This did not take long. I spent some time looking through a high school geometry text book as well as notes from an advanced mathematics course that I had taken two years earlier where we proved some basic geometric proofs. What Jessica had done informally, proving that triangles with equal areas are not necessarily congruent, was not a trivial piece of mathematics and one that many students find difficult (Chazan, 1989).

A New Problem to Solve

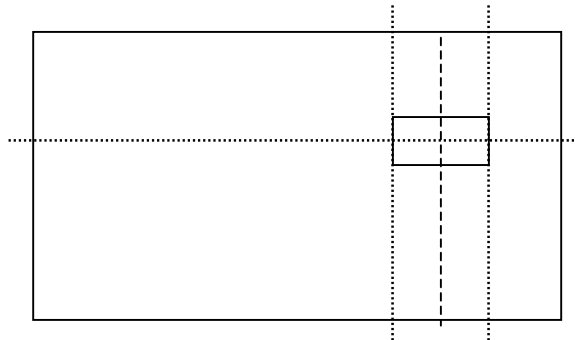
After sharing that I had been able to prove that all four pieces in Jessica's brownie were all equal shares, my instructor shared another interesting brownie problem that she had heard.

Here is a pan of brownies with one rectangular brownie cleverly removed by my hungry little brother.



Is there a strategy for splitting the pan of brownies into two equal shares now that one piece is missing? If so, what happens if I limit the number of cuts? What happens if the missing brownie piece is in different locations in the pan?

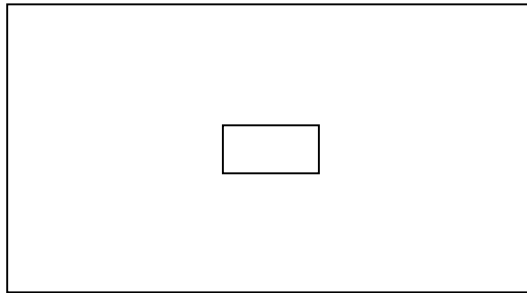
I was intrigued by this new problem and happy to still be working with mathematical chocolate. After thinking about the problem for a while, I decided that there could be several solutions to splitting up the pan of brownies equally. Most of these solutions required several cuts. For example:



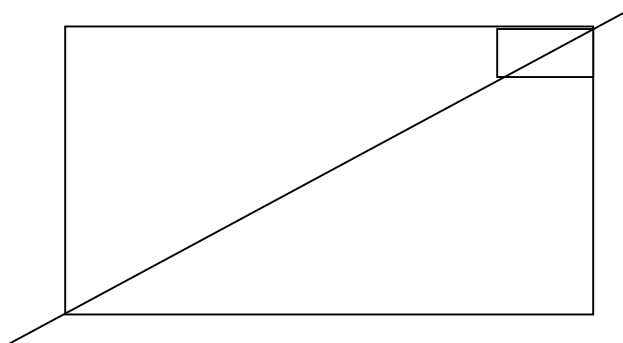
I was curious about being able to do this with one cut. One way that I discovered, although impractical and somewhat challenging (in a culinary sense), would be to cut the pan of brownies through the middle horizontally. Then, each person would have a really thin brownie that looked exactly alike, hole and all. It seemed as if I had exhausted all the work I could do on paper so I decided to use an interactive geometry software to assist me in my exploration. Using Geometer's Sketchpad turned out to be a profitable decision. However, there was a rather large investment of time to become familiar with the tools and options available. Learning to navigate through the program interface and understand the specific tools available was time consuming. One of the best benefits of using the software was the automatic calculate command. This saved me a lot of time and I could be confident that the calculations were exact.

I started to consider which location the missing brownie piece to be in to make the one cut the "easiest". I figured that the best place for the missing brownie piece to be

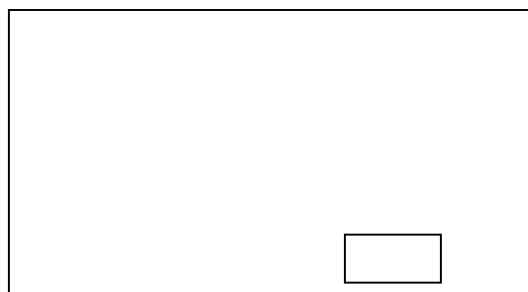
located would be the very center of the pan of brownies so that if I cut exactly through the middle of the pan, the missing brownie piece would also be divided equally between the two sides.



But what if the missing brownie piece was located somewhere else in the pan? It seemed that the next “easiest” location to place the brownie would be in a corner. The cut from corner to corner divides the pan and missing brownie piece equally:



But I knew that the most difficult location was still to be solved. What if the missing brownie piece was located in some non-trivial position in the pan?



When it came time to think about this possibility, I found that using the previous examples really helped me out. It seemed in both cases that the cut went through the center of both the pan of brownies and the missing brownie. So I tried it with randomly located missing brownie pieces and it worked! It took a lot of labeling and measuring in GSP but it helped me to calculate and compare areas. I had discovered that regardless of the location of the missing brownie piece, only one cut was necessary to divide the pan equally.

After solving this challenge, I was encouraged to ask another question and attempt to solve it. I decided once again to stick with chocolate. My new problem was based upon the previous problem. I wanted to know if there would be a strategy to divide the same pan of brownies (the one with a missing piece) into fourths using only two cuts. There seemed to be an infinite number of wrong paths. The time was painstaking as I went down each path only to find out that I wasn't getting anywhere. The exploration with a software application that I was unfamiliar with seemed at first a hindrance but later became critical to the investigation. It would have taken forever to draw and check the areas of each shape for the cuts that I was trying. I tried making cuts based on the location of the missing pieces and the first cut. I drew circles around the midpoints of the pan of brownies and the missing brownie and tried to find a connection between some of those shapes. The list of attempts and defeats continued on and on. The most difficult aspect of trying to solve this particular problem was that the longer I attempted different strategies, the more it seemed that I was pursuing a question that seemed to have no solution.

My exploration with the problem ended because of the time constraints of the semester but I look forward to the day that I am afforded an opportunity to revisit the brownie problem. I am still curious about the results I may find and the new questions that may emerge by continuing to explore the problem.

So Why Bother?

Elementary mathematics classrooms provide rich mathematical opportunities for students as well as teachers. Reflecting upon this classroom episode and the recent opportunity to return to the mathematics has been a powerful example of what NCTM claims as, “doing mathematics.” Through the verbs of doing mathematics such as, exploring, investigating, conjecturing, constructing, discovering, and justifying, I was successful at solving one of my explorations. Although I was not successful at finding a solution for the second exploration, the experience was just as beneficial. I look forward to engaging in additional explorations that will continue to improve my own mathematical understanding and my ability to become an effective teacher of mathematics as envisioned by the NCTM.

CHAPTER THREE

Methodology

Methodology has roots in the Latin *methodus* meaning “a way of teaching or going,” and in the Greek *methodus* meaning “scientific inquiry, method of inquiry. These roots can be traced back to their original meaning of “pursuit, following after” from *meta* meaning “after” and *hodos* meaning “a traveling, way”. (<http://www.etymonline.com>). I look to these etymological meanings to help recount a method of coming to understand. This chapter reports on the “after way” of the study.

The purpose of this study was to understand the mathematical experiences of mathematics education students enrolled in a doctoral program. The following questions guided the study:

1. What is the nature of the mathematical experiences that mathematics education doctoral students identify as components of their program?
 - What is the meaning of these experiences in the context of our doctoral student careers?
 - How are these experiences situated our preparedness as they enter the mathematics education field?
2. How can specific mathematical experiences influence the identity and preparation of a mathematics education doctoral student?

A qualitative approach was chosen to obtain a rich data set to describe the mathematical experiences of mathematics education doctoral students. This included conducting semi-structured interviews and investigating written data sources. Results

were obtained through the use of qualitative analysis procedures. The researcher searched for themes and patterns that would provide insight into the questions asked.

This chapter begins by illuminating the researcher's journey to becoming a qualitative researcher. The chapter continues with a description of the methodology of the study utilized by the researcher to uncover understanding of the research questions. A detailed account of the procedure and analysis, as well as a description of participant selection, data sources and methodological rationale is then provided.

Attributes such as personal history, biography, and lived experience influence researcher's values, biases, and beliefs and in turn influence data collection and analysis (Creswell, 1994; Denzin & Lincoln, 1994). Qualitative research, an interpretive research method encourages the researcher to acknowledge personal values, biases, and beliefs. Thus, I begin this chapter by continuing to share my journey of methodological knowing and being.

Developing as Mathematics Education Researcher: A Look Inward

Human Science: Turning towards a Qualitative Understanding

"...human science research is itself a kind of *Bildung* (education) or *paideia* (upbringing of a child); it is the curriculum of being and becoming (van Manen, 1990, p. 7).

In my own mathematical upbringing, I've had a sense that we do mathematics because it is a part of the curriculum. Mathematics was a subject, an hour of the day, a text, a lecture. It was not a way of thinking or being. It became more evident upon teaching children and getting excited about helping to provide rich meaningful mathematical experiences that my own mathematical upbringing seemed to be weak and fragmented.

My experiences in a particular mathematics class (Foundations of Mathematics or FM) powerfully affected my ideas of what teaching and learning mathematics can look and feel like. My conceptions of how a mathematics classroom community should behave also were affected. It is my reflection upon these experiences that has pushed me to consider what is important in a mathematics learning environment.

In the same way that the FM class was experienced in a different way from the traditional learning in a mathematics class, I choose to seek to uncover these experiences by drawing upon a research methodology that is different from more traditional forms of mathematics education research methodologies.

We explain nature, but human life we must understand. Whereas natural science tends to taxonomize natural phenomena (such as in biology) and causally or probabilistically explain the behavior of things (such as in physics), human science aims at explicating the meaning of human phenomena (such as in literary or historical studies of texts) and at understanding the lived structures of meanings (such as in phenomenological studies of the lifeworld) (Dilthey, 1976, as cited in van Manen, 1990, p. 4).

To understand my phenomenon, I turn to human science. Where natural science studies objects, things, and the way these behave; human science studies persons or beings that have consciousness and who act purposefully in and on the world (van Manen, 1990). The preferred methodologies for natural science traditionally have been detached observation, controlled experiment, and mathematical or quantitative measurement. When textual data are quantified, the goal of understanding a phenomenon from the point of view of the participants and its social and institutional context is lost (Kaplan & Maxwell, 1994). Therefore, the preferred method for understanding and researching human science involves description, interpretation, and self-reflective or critical analysis.

On this journey of understanding while being and becoming a researcher, I am guided by tenants of human science researchers who have paved the road before me:

1. There are many ways to come to know something, and even then, such knowing is partial.
2. There are numerous ways for us to report.
3. All of our messages have agendas - personal, political, gendered, racial, ethnic.
4. Our language creates reality.
5. As researchers we are deeply interrelated with what and who is being studied.

Research is context-culture bound. So is writing.

6. Affect and cognition are inextricably united.
7. What we understand and report, as social reality is multifaceted, sometimes clashing, and often in flux.
8. We cannot say that narrative reflects “the” reality. We can say that with the help of the reader, narrative creates a version of reality (Ely et al., 1997).

A Look Inward: Apprehensive of Research

In this post-modernist era, there are numerous approaches to “knowing”. Researchers have never before had so many strategies and methods of inquiry from which to choose. Phenomenology, a way to study the lived meaning of an experience, is a relatively new option for researchers in mathematics education. By borrowing from the phenomenology tradition, I hope to uncover the lived meaning of the mathematical experiences of mathematics education doctoral students and provide a deep rendering of the way we experienced and understood them.

At the beginning of my doctoral student career, I had little interest in engaging in research, and the thought of dissertating was off the radar screen in terms of one of my goals for doctoral study. This knowledge of research and continued experiences in research in the field slowly evolved in to something I had interest in, although I was not convinced how it would fit into my future. I declared myself “qualitative” almost immediately, being drawn to the personal interaction and nature of qualitative research. I found the quantitative courses confusing, cold, and contradictory to the pillars of the education field that attracted me. We would spend hours reading and discussing the technical aspects of quantitative statistics and spend our out-of-class time practicing statistical techniques by hand. I struggled to see how these things helped me to understand my “participants”. The humanness I loved about teaching and learning was void in the quantitative world.

The qualitative courses I took were more attentive to the kinds of details that speak to the heart of teaching and learning. I admit, however, a sense of both wonderment and uneasiness as I attempted my first few qualitative papers and studies. I was not confined to a set of rigid and prescribed steps. Although freeing, I worried that I wasn’t “doing it right”.

At the same time I was enrolled in the FM course, I was enrolled in another pivotal class. It was in the same serendipitous semester that I was introduced to phenomenology. My inner most feelings about why bothering to conduct research were being answered in parallel with my other bothering questions. Finally, I felt I was able to engage in doing meaningful research about the learning and experiences that occurred in this mathematics class.

As researchers we all draw on the reactions to our work by a range of audiences. It is worth us reflecting the ways in which this mirrors the process of learning mathematics. Do our learners interpret our responses as mathematics teachers in a similar way? Do they draw up a set of criteria which they feel they must conform to if they are to be successful learners of mathematics? If they cannot conform to this set of criteria do they define their identity in opposition to these criteria thus defining themselves as ‘not mathematician’ (Cotton, 2002, p. 10).

How are the FM experiences in mathematics related to my experiences in engaging in phenomenological study? As a learner of mathematics, I relied on a traditional set of criteria, engrained in me from a very young age, of what it meant to do mathematics, what it meant to ask mathematical questions, and what it meant to be a student of mathematics. In FM, these criteria had little to do with my prior “mathematician-ness”. New criteria emerged about mathematics and learning mathematics. Similarly, as I met phenomenology and began a journey into knowing, my identity was being transformed from “not researcher”.

Research Design

I used features from the phenomenological inquiry tradition to guide this study. This type of investigation resulted in rich descriptions of the perceptions of the participants (Willis, 1991). After each participant shared descriptions of their experiences, the researcher was able to establish the tenets for data reduction followed by data analysis. The data reduction and analysis strategies were borrowed from the descriptive case study research design.

Phenomenology

I chose to shape the way I engaged with my questions by following a hermeneutic phenomenological approach. By using phenomenology, a way to study the lived meaning of an experience, I was able to create a deep rendering of the way we experienced and

understand the courses and uncover the lived meaning of the participants' mathematical experiences.

Human science incorporates several perspectives, philosophies, and methodologies. As a philosophy and methodology, phenomenology seeks to uncover how human awareness is implicated in the production of social action, social situations and social worlds (Natanson, 1970). By examining important issues through a phenomenological perspective, an opportunity is created for others to experience multiple vantage points in order to understand others' experiences.

The term, phenomenology, is derived from two Greek words: “phainomen”, the phenomenon, and “logos”, the activity of giving an account (giving a logos) of the various ways in which things can happen (Moran, 2000). The *American Heritage Dictionary* (2000) describes phenomenology as “a philosophy or method of inquiry based on the premise that reality consists of objects and events as they are perceived or understood in human consciousness and not of anything independent of human consciousness.” Further, the *Merriam-Webster Medical Dictionary* (2002) defines it as “the way in which one perceives and interprets events and one's relationship to them in contrast both to one's objective responses to stimuli and to any inferred unconscious motivation for one's behavior”.

...hermeneutic phenomenology is a philosophy of the personal, the individual, which we pursue against the background of an understanding of the evasive character of the *logos* of *other*, the *whole*, the *communal*, or the *social*. Much of educational research tends to pulverize life into minute abstracted fragments and particles that are of little use of practitioners. (van Manen, 1990, p.7.)

The job of a phenomenologist is to attempt to most accurately report the lived experiences of a group of individuals as a way to understand the meaning and essence of

human experience (Moustakas, 1994). This is done by capturing the individual's perceived realities of a particular context and describing them through writing (Hopkins, 1994; Moustakas, 1994; Patton, 2002). Under the phenomenological approach of inquiry, the purpose of this research was to describe and interpret the qualities of the participants' experiences in their mathematics courses and provide a description that captures the essence that exists in our collective experiences.

Descriptive Case Study

Descriptive case studies in education present a rich and detailed picture of a phenomenon or problem situation. According to Merriam (1988), descriptive case studies have been particularly useful in education because they can highlight areas where little research has been conducted, such as innovative programs and practices. Since there is little research regarding the mathematical experiences of mathematic education doctoral students, I also chose to make use of aspects of descriptive case study design.

Following descriptive case design methods, a reduction of data to salient features was the initial step in the analysis and interpretation of the data. This interpretation of the salient features of the data allowed me to describe and analyze patterns or relationships among the data (Miles & Huberman, 1994). The themes emerged from the data and were used to support or refute the interpretation of patterns making the study inductive in nature.

A Marrying of Methodologies

I found the use of two approaches important in the asking and researching of my questions. Phenomenological and descriptive case study approaches are well-suited and complementary for several reasons. Merriam (1988) stated both phenomenology and case

study allows the researcher to investigate intricate social environments in detail. Both types of inquiry provide holistic and descriptive accounts of a phenomenon and typically report events as they unfold. In both frameworks, the researcher does not manipulate variables in order to prove a hypothesis. Finally, both approaches tend to present data about a phenomenon from the perspective of several individuals with the purpose of describing the complexity of a specific encounter or event.

I chose to use descriptive case study and phenomenological approaches for this study. The borrowing from these traditions was a fluid occurrence but reflection upon my choices has helped clarify where I was more dependent on one over the other. The phenomenological tradition helped me to conceptualize the object of study, think about the phenomenon, situate the problem, and gather rich evidence and contextual description. I then looked to descriptive case study tradition to advance understanding of the phenomenon through establishing the bases for data interpretation, identifying patterns within the data, developing assertions or generalizations about the cases, and identifying alternative interpretations of the results of the study to be further pursued (Stake, 1994).

Writing: An Interpretative Research Tool

As I prepared to understand the experiences in FM, both mathematically and pedagogically, I engaged in writing to communicate, understand, connect, engage, and deepen. As Richardson (2000) explains, “I write because I want to find something out. I write in order to learn something that I did not know before I wrote it (p. 924).”

All research involves writing. In quantitative research, a large part of the writing occurs when the researcher is writing up the research report. The purpose of the research

report is to communicate to others what he or she has done. These reports typically follow the recipe of introduction, literature review, method and methodology, results, and implications. In human science, and most qualitative research paradigms, the final text also communicates what has been done and learned. But, in many cases, it is much more than a description of methods and steps taken. The writing is, in itself, part of the research, part of the data, and inseparable from research itself (Richardson, 2000). According to Richardson (2005), “qualitative work carries its meaning in its entire text (p. 960)”.

In standard social scientific discourse, methods for acquiring data are distinct from the writing of the research report, the latter presumed to be an unproblematic activity, a transparent report about the world studied. When we view writing as a *method*, however, we experience “language in use” (Richardson, 2000, p. 960).

Elizabeth Adams St. Pierre describes writing as such: “Writing is thinking, writing is analysis, writing is indeed a seductive and tangled method of discovery” (Richardson & St. Pierre, 2005, p. 967). This research will embrace *writing as a method* of inquiry, not as merely a methodological measure to complete a research project but instead as a way of knowing, a method of discovery and analysis (Richardson, 2000. p. 923).

I consider writing a method of inquiry, a way of finding out about yourself and your topic...a method of discovery and analysis. By writing in different ways, we discover new aspects of our topic and our relationship to it. Form and content are inseparable (Richardson, 1994, p. 516).

Participants: Who Were the Others?

This study sought to understand the mathematical experiences of mathematics education doctoral students. The overarching pool of potential participants available included all mathematics education doctoral students enrolled in the mathematics center

for teaching and learning at the university in which the researcher was enrolled. I chose to narrow the pool by only considering participants who had finished doctoral coursework therefore maximizing the number of mathematical course experiences that a participant may have. The participants were either engaged with dissertation research work or completing comprehensive exams at the onset of this study.

As described earlier, a particular mathematics course had been the impetus for this study. The uniqueness of the course and the experience it afforded its students provided a comparison for discussion. Therefore, I chose to include participants who had taken the FM course, as well as participants who had not taken the course. Three of the participants were students in both semesters of the FM course, one participant was a student in one semester of the FM course, and two participants did not take the course. The researcher utilized purposeful sampling as a way to best select “information rich participants” (Patton, 1990, p. 169) who can “best help us understand our phenomenon” (Creswell, pg. 193a). Engaging in purposeful sampling (Merriam, 1991) helped to ensure that I would gather relevant information and adds to the trustworthiness of the research because the participants will be most suitable for the questions I am seeking to understand.

Although the pool of six participants had varying backgrounds of mathematics courses in their academic histories, they shared several common characteristics. The participants, all women, had been K-12 teachers earlier in their careers and had made the decision to leave teaching and pursue doctoral work in mathematics education. Each participant was enrolled as a mathematics fellow in the mathematics center for teaching and learning and therefore had taken several of the same core courses required by the

center's policy. In addition, the participants shared similar graduate experiences such as teaching undergraduate courses and planning and facilitation professional development. According to Greenbaum (1998), a homogeneous mix of participants is desirable as "the more homogeneous the group is, the better the participants will relate to each other and the higher the quality of the input they will generate" (pg. 62). A homogeneous group will "spend less time explaining themselves to each other and spend more time discussing the issues at hand" (Morgan, 1997b, p.59).

The identities of the participants of this study have been protected by the use of pseudonyms. The pseudonyms are used throughout the study. Findings from this study did not impact course grades or evaluation. Study results were not intended to be used to evaluate instructors.

Data sources

How do we collect an experience? How do we collect feelings, thoughts, and essences? What is it, that I call data? In the human sciences, data collection can very often include a wide range of sources: interviews, surveys, written and verbal reflections, personal journals, written work, and observations.

According to the *American Heritage® Dictionary* (2000), the word "data" is the plural of the Latin "datum", and means "something given". What were the "things" given to me in this research? I was given permission to watch videotaped class sessions, read classmates journals and reflections, and examine other class artifacts. The most powerful thing I was given, however, were the words given, both spoken and written, during conversations and communication with participants. These gifts of conversation provided me with the textual sources used to reach a deeper understand of the fundamental nature

and essence of our experiences.

The Spoken Conversations

Conversation is a process of coming to an understanding. Thus it belongs to every true conversation that each person opens himself to the other, truly accepts his point of view as valid and transposes himself into the other to such an extent that he understands not the particular individual but what he says (Gadamer, 1960/1989, p. 385).

My primary role as researcher during conversations with participants was to provide a context in which participants felt free to “fall into conversation” and describe their experiences in detail. These experiential conversations represented a risk on the part of the researcher as I relinquished a substantial amount of control during parts of the conversations. However, the payoff of letting go is the uninhibited conversation that occurs providing the researcher with a more robust narration of experience, thoughts and feelings. In the conversations that occurred, I primarily provided an opening question, prompt, or scenario and allowed the conversations to flow. This was intended to, “yield a conversation, not a question and answer session” (Thompson et al., 1989, p.138).

The conversations with my participants occurred in three stages. Conversations were tape-recorded and transcribed. The text from each transcription helped me engage in each subsequent conversation. The first stage, consisted of individual conversations with my participants, and allowed me to initiate dialogue surrounding participants’ personal stories and experiences in the course. The second stage involved gathering my participants together as a group to discuss the ideas and themes that emerged during the first stage. For example, during this conversation I shared a list of descriptive words that participants had used during the previous conversations that described their mathematical experiences. The final stage also consisted of a group discussion.

Throughout the conversations, I encouraged participants to focus on how events or experiences made them feel or what certain events were like. For example, I asked questions such as, “How did it feel to be a learner in that class?” I asked participants to focus on specific examples and provide detailed accounts of individual episode that felt particularly powerful.

The Written Conversations

Interpretation, in the sense relevant to hermeneutics, is an attempt to make clear, to make sense of an object of study. This object must, therefore, be a text, or a text-analogue, which in some way is confused, incomplete, cloudy, and seemingly contradictory - in one way or another, unclear. The interpretation aims to bring to light an underlying coherence or sense (Taylor, 1976, p. 153).

In addition to the transcribed conversation as textual sources from which to draw, I also communicated with participants through written conversations. That is, in the course of our conversations and my writing, opportunities arose for me to request written reflections or reactions to what I had written. These written responses were in reaction to written prompts, questions that were raised, excerpts from course journals or video segments. This back and forth written conversation provided more textual sources from which to draw.

Analytic notes (Ely et al, 1991) were used in order to help move along data collection and data analysis. Analytic notes are conversations with one’s self about what is occurring in the study. These notes are written entries about the data collection and analysis that they themselves become part of the data.

Procedure

Although, this study reports data from three formal series of interviews, data collection has been an ongoing activity, consciously and unconsciously for several years. The individuals in this study are more than research subjects. As fellow fellows to the researcher, we are peers, colleagues, and have naturally developed friendships with each of the participants through the journey of becoming. As a researcher, this has had its advantages and challenges. The challenge has been to respectfully maintain a researcher-subject relationship to a certain degree during the duration of the study. The advantages have included an extreme willingness by the participants to participate in the study and willingness to share above and beyond the confines of the three “formal” interviews conducted. This includes multiple phone conversations and email communication. Participants were willing to add missing information and answer additional questions related to the study. The other advantages included simply knowing my participants stories well and in some instances even living through common experiences as we have traveled together.

Conversations

Stage I.

The first round of formal conversations was conducted individually during one week in July of 2006. These interviews were intended to gather background information on the mathematics pedigree of the participants. Participants were asked to speak about their mathematical histories. Participants expounded on “how they came to doctoral work in mathematics education.” Each participant shared the story of the kinds and nature of mathematics courses they took during their undergraduate and graduate programs and how their teaching experiences fit into their backgrounds. Finally, participants were

asked to focus in on the mathematics experiences that occurred while in their doctoral programs. In some interviews, participants referred to their transcripts and talked about each class in the order taken while other participants preferred to speak about the courses more organically after they had identified which courses they had taken. Participants were encouraged to describe their roles and experiences as students in the classes, the role of the instructor and to reflect on the nature of mathematics in the course.

Following the interview, the conversation was transcribed. There were some informal conversations that occurred in-between the first and second interviews that allowed me to fill in some of the holes in my understanding of their experiences. These conversations occurred via email and phone. The seeking of additional and explicitly relevant data, according to Erickson (1986), “deliberately enable and empower intuition, rather than stifle it (p. 140).”

Stage II.

The second stage of conversations occurred in September of 2006. This was a group conversation and all six participants were present. The session was held in the home of one of the participants on a Friday afternoon and lasted approximately 2 ½ hours. To help establish a comfortable environment, refreshments were served. The beginning of the afternoon was spent briefly catching up and sharing news. After this informal talking, I focused the conversation on the study. I asked the participants to imagine a fictitious university, Utopia University, which embodied the best of all they had learned with respect to mathematics education, teacher education, and mathematics. The purpose of the use of Utopia was to provide a common context for which to compare experiences. I asked the participants to discuss their ideas of what knowledge, skills, and

experiences would be needed to become mathematics education faculty at Utopia University. Participants were then asked to discuss which of these important features they felt they had ownership of themselves. Participants shared the strengths and weaknesses in relation to the essential features and discussed where in their doctoral programs they felt they had acquired such skills.

Participants were then asked to think about the mathematics education doctoral students that might be enrolled in Utopia University's doctoral program. The researcher asked the participants to share the kind of descriptions they would want these students to share about their mathematical experiences. Participants were asked to specifically think about the teacher's role, student's role, the mathematics, and the discourse. These lists were shared out loud and discussed. Then, I read a list of descriptors that was generated from the first formal conversation with participants. The descriptors were the words that participants used to describe the nature of their own mathematics courses in their graduate programs. (see Prologue).

Stage III.

The third conversation occurred in October of 2006. This was a group conversation and it took place in the home of another participant. All participants were present for this three-hour conversation. Again, to help promote a warm and inviting environment, food and drink were provided for participants as they engaged in conversation. The session was divided into two sections. Participants watched a video clip from the FM class. I read the following prompt:

We will start today with watching some video clips from the FM course. The structure of the course was such that the instructors would pose a question, problem, scenario to the group and we would be given time to think about it, ask questions, ask what if, why, etc. We would share our questions publicly- on the

board and set off to work and ask some more questions together, with a partner, individually but always reconnecting with the whole group throughout the class.

I've identified one such scenario that was posed in our class. This occurred mid-semester of the second semester of FM. I want to watch it together for a couple of reasons. First, I'm hoping for us to talk about and do some mathematics inspired from the mathematics we see. I'm looking for us to try and understand the mathematics that we did but also to get your reactions and questions to the mathematics we did -perhaps thinking about where we went with the mathematics and how we got there.

I stopped the video after three previously decided upon scenes to allow the participants to “do the math”. I paired up the participants into strategic couples: one person who had taken two semesters of FM with one person who had not taken FM at all or had taken only one semester. This was a conscious choice hoping that the person who had taken FM could provide the mathematical and contextual background for some of what occurred in the video. The doing of the mathematics together provided a natural way for participants to share their mathematical experiences and compare them to those they were witnessing and/or experiencing in the video session.

During the second section, participants were given a list of the Utopia descriptions that was generated during the previous conversation. I asked the participants to think about the video they had watched and mathematics that they had engaged in and discussed and compare them to the Utopia descriptors.

With a partner- think about the video we watched- then go through our list from Utopia U and see if you can think of evidence that you observed these things were occurring. Be as specific as possible- write down your thoughts, comments, ideas...

Participants worked independently and then in pairs to discuss their thoughts. Then each pair took turns sharing what they had discussed. In this whole group

discussion, participants shared what they had witnessed in the video, described visions of Utopia, and recalled past mathematical experiences.

Other Data Sources

During both semesters of the FM course, students were asked to keep journals of their mathematical thinking, questions, and experiences. These journals were available to me and were rich sources of textual data. Most FM class sessions were videotaped. These videos were also available to me as sources of data.

Analysis: How Did I Engage With My Data?

In engaging with this research, I became the data processing instrument. The analysis of data occurred with words, texts and conversations in phenomenological studies. Interpreting these texts occurs on several levels. As primary author, my reading of the text is one interpretation; however, phenomenology acknowledges that all other readers of the text also become authors as they bring their own knowledge, ideas, values and set of experiences to the text.

As primary researcher and research tool, a starting point for this research was through personal reflection on my experiences in FM and other mathematics courses taken. This included current reflections as well as returning to past renderings of the experiences.

After reading and reflecting on a text, a search for emerging themes occurred. As van Manen (1990) states, the themes represent the “knots in the webs of our experiences, around which certain lived experiences are spun and this lived through meaningful wholes” (p.90). The analysis of data was a fluid, ongoing process. Because I was driven to engage with these questions due to my own experiences and the knowledge of

experiences of others in the mathematics education doctoral program, data analysis began early in the study.

Because of the personal nature of this study to the researcher, it was critical to attempt to maintain an objective perspective, to the extent possible, of the data. That is, I paid particular attention to ensuring I was hearing and interpreting participants' perspectives correctly. One way to help control for this was to engage participants in group conversations where the researchers' voice was more of a fellow participant than of a researcher.

The formal analysis of data, that is, the analysis that has occurred specifically surrounding the identified data sources, has also been fluid. Prior to the first interviews with participants, I had access to the following data sources from which to draw: course documents, course videos, student work, student journals and reflections, as well as my own reflections and experiences. These pieces of data helped me to identify the nature of my questions in the first interview.

After each of the interviews in the first round, I transcribed the conversations and took note of key pieces of information. The information that participants shared about the nature of their mathematics coursework was combed through for descriptive words and phrases. These words and phrases were then used for a portion of the discussion in the second conversation.

Data analysis was approached from an interpretive perspective. A coding system, as described by Patton (2002), Bogdan and Bilken (1998) and Berg (2001) was established. The data from both written and spoken sources determined the coding categories.

Coding Procedures

Copies of each transcript were produced. The first step was an “open coding” process. The first interview focused on participant’s mathematics histories; consequently, the first code that appeared was “experience” (coded as “A”). The passage was highlighted with a yellow marker and a list of codes grew. Subsequently, the entire transcript was analyzed for any reference to “Experience” and each appropriate passage was cut out. The same transcripts were then analyzed for a second code such as “Future”, using a green highlighter. Each code and highlighting color was recorded on the transcript itself. The transcript from conversation #2 was then analyzed. During the analysis of conversation #2, new codes emerged from the text. When additional codes emerged from the transcripts of conversation 2, the researcher returned to the transcripts of conversation #1 to analyze. This iterative process continued until all three conversations were thoroughly examined. The data from the three conversations provided a form of triangulation. That is, the researcher found that these codes were present throughout data collection as participants spoke of their experiences. Consequently, a set of follow-up questions targeting “purpose” for example, was sent via email to participants. The data from the responses was coded, as described above and included into the study.

The highlighted comments were cut of the transcripts and placed into coding bins. By arranging and rearranging initial codes and subsequent data, themes and patterns in the data emerged. Erickson (1986) The summarized the analysis of the data:

In conducting such analysis and reporting it, the researcher’s aim is not proof, in a casual sense, but the demonstration of plausibility...the aim is to persuade the audience that adequate evidentiary warrant exists for the assertions made, the

patterns of generalizations within the data set are indeed as the researchers claims they are. (p. 149).

The themes were organized into two cases. A description of the organization of these cases is included in the following chapter. After each case was written, participants were asked to read and engage in member checking of the sections that pertained to them. This was done to ensure that I had accurately presented the essence of participants' experiences and comments. Throughout each stage of writing, participants provided their comments and clarifications about what I had written. These comments were then used to more accurately represent participants' experiences in the writing.

Summary

This chapter has provided the reader with the methodology in this study. First, I communicated my background and coming to understand a qualitative research approach. Second, I defined the interpretive research method guiding the study. Third, I shared how I collected and analyzed data through individual and group conversations, student journals, and other textual sources which were created as descriptive cases. The next chapter reports on the details of the data analysis and case presentations.

CHAPTER FOUR

Introduction

This study examines the mathematical experiences of mathematics education doctoral students. In addition to sharing my own experiences, I draw upon the experiences of others who have shared the familiarity of being mathematics education doctoral students. As the primary research instrument, my perspective, beliefs, and understandings have influenced the literature I have chosen to draw upon, the nature of data collection, and the ways I saw themes emerge from the data.

This chapter presents three distinct yet related sections that lay groundwork for the two data chapters on course experiences. The first section introduces the reader to each of the six participants in the study. In the second section, I present a brief description of the major themes that emerged from the data. I start each theme description with a working definition created from multiple sources. To further illustrate the theme, I share clips of textual data from conversations, class journals, video transcripts and other communication in an attempt to clarify and support the definition, and provide examples of the theme. These descriptions include brief academic and mathematical histories. Finally, I establish the framework and rationale for the two data chapters that follow.

Participants

This is a study that seeks to understand the mathematical experiences of mathematics education doctoral students. The overarching pool of potential participants available to the researcher included all mathematics education doctoral students enrolled in the center at Maryland. I chose to narrow the pool by only considering participants who had finished coursework therefore maximizing the number of mathematical course

experiences that a fellow may have had. As described earlier, a particular mathematics course has been the impetus for this study. The uniqueness of the course and the experience it afforded its students provided a rich comparison for discussion. Therefore, I chose to include participants who had taken the course, as well as participants who had not taken the course. The participants are all ABD (All But Dissertation): three of whom were students in both semesters of the foundations of mathematics course, one of whom took one semester of the course, and two of whom did not take in the course.

Wendy

Wendy completed a degree in Early Childhood Education from a small state school and taught elementary school for many years. She returned to school to pursue a Master's degree in Mathematics Education at a large Midwestern university.

Wendy began her PhD program in the fall of 2001. During her coursework, Wendy took Calculus I and II, Transitions to Advanced Mathematics, Abstract Algebra, Foundations of Mathematics I, and the required statistics series for education students. Wendy describes her interests, knowledge, and experience as being focused on elementary mathematics education.

Mia

Mia studied secondary mathematics education at large private university located in the mid-west and graduated in 1997. The mathematics courses Mia took included Calculus II and III as well as Abstract Algebra. After graduation, she began teaching high school mathematics. She taught Pre-Algebra, Algebra, and Calculus.

Mia enrolled in the graduate school at a large public university located in the mid-Atlantic region of the U.S. and finished her coursework in 2001. Her mathematics

coursework included Linear Algebra, Number Theory, and Statistics. Mia began the mathematics education doctoral program in the fall of 2001. She was a part of the first cohort of students in the center. The mathematics courses that Mia completed during her doctoral coursework included Foundations of Mathematics I and II, Combinatorics, and Statistics. Mia has knowledge and experience in teaching high school mathematics education but also has interest in elementary mathematics education.

Faith

Faith attended the same university for her undergraduate and graduate degrees. She began her undergraduate career with an intended major of engineering. Thus, she was slated to take several mathematics courses including three calculus courses. She changed her major mid-way through the engineering program to elementary education and graduated in 1996. After graduation, she began teaching elementary school.

During her graduate program, Faith was required to take Abstract Algebra, Number Theory, and Statistics. She graduated in the spring of 2001 with a Masters degree in mathematics education. Faith began the mathematics education doctoral program in the fall of 2001. She was a part of the first cohort of students in the center. The mathematics courses that Faith completed during her doctoral coursework included Transitions to College Mathematics, Abstract Algebra, Foundations of Mathematics I, and Statistics. Faith describes her interests, knowledge, and experience as being focused on early childhood/elementary mathematics education.

Kristen

Kristen attended a small liberal arts college and completed a double major in Mathematics and Elementary Education. While there, she took an extensive list of

mathematics courses including Math for Elementary Teaching, Calculus 2 and 3, Linear Algebra, Geometry, Modern Algebra, Graph Theory, Operations research, History of Math, and Proof and Structure.

After completing her undergraduate degree, Kristen began teaching sixth grade mathematics. While teaching middle school, Kristen also pursued and completed a Masters Degree in Mathematics from a large private university located in the mid-Atlantic region of the U.S. This degree required Kristen to take more mathematics courses including Analysis 1 and 2, Modern Algebra 1 and 2, Geometry, History of Math, Topology, Statistical Methods, and Regression. Kristen began doctoral courses in the fall of 2003. During her doctoral coursework, Kristen took Foundations of Mathematics I and II, Number Theory, and the required statistics series for education students. Kristen describes her interests, knowledge, and experience as being focused on middle grades and elementary mathematics education.

Eva

After high school, Eva attended an Ivy League school where she studied History. The only mathematics class she took in her undergraduate program was Calculus II. Eva graduated in 1994 and attended a large public university for her masters program in mathematics education with a focus in secondary mathematics. She took an extensive list of mathematics courses including Euclidean and Non-Euclidean Geometry, Analysis, Number Theory, Abstract Algebra, Differential Equations, and Linear Algebra. After graduating in 2000, Eva taught high school mathematics for three years. The courses she taught included AP Calculus, Reform Calculus, Advanced Mathematics and Differential Equations.

Eva began her doctoral work in the fall of 2002 and continued to take Biostatistics, Dynamics and Chaos, Foundations of Math I, Foundations of Mathematics II, Philosophy of Mathematics and the statistics sequence for Education Students. Eva describes her interests, knowledge, and experience as being focused on high school and post-high school mathematics education.

Anne Marie

I attended a large Midwestern university and graduated with a degree in curriculum and instruction. Immediately following completion of my degree, I began teaching third grade. I started graduate work in the fall of 1998 in curriculum and instruction with an elementary mathematics education focus. I completed this degree while continuing to teach. In the fall of 2000, I did not return to teaching and instead began the doctoral program. I took the following mathematics classes in my doctoral program: Calculus I and II, Transitions to Advanced Mathematics, Abstract Algebra, Foundations of Mathematics I and II, and the required statistics series for education students. My interests, knowledge, and experience are focused on elementary mathematics education.

The participants in this study are all female, providing a set of experiences that is uniquely female. However, the participants represent a range of racial and ethnic diversity. Participants' backgrounds were similar in that all six participants had taught at some point in their careers. However, the range of teaching of experiences and mathematical histories varied. It was through interacting with these five participants and the rich set of data including courses journals, video, and conversations that I was able to identify the themes in the data.

Themes

After the writing of personal experiences and the collection of others' experiences, the hermeneutic phenomenological tradition encourages tracing etymological sources of language, searching idiomatic phrases, exploring experiential descriptions in literature, poetry, and other art forms related to the phenomenon, and consulting phenomenological literature as other sources of insight and possible meaning (van Manen, 1990). After reading and reflecting on text, a search for emerging themes will occur. As van Manen (1990) states and was quoted earlier, the themes represent the "knots in the webs of our experiences, around which certain lived experiences are spun and this lived through meaningful wholes" (p.90). Several themes emerged from the data. After spending considerable time reading and analyzing data, the following three themes emerged: authority, interaction, and mathematics. As I engaged in writing and further data analysis, it became evident that a fourth theme, sense of purpose, was also significant to this study.

Although all four of the themes appeared in each of the participants' conversations and reflections on their experiences, the themes were not equally represented among participants. For example, the theme of authority came up more often than purpose for some of the participants, thus suggesting authority was more influential in thinking about the experience than purpose was. I believe this variation with respect to themes across participants provides one way of acknowledge that I, the researcher, was able to listen carefully to participants voices and represent their perspectives without my own experience influencing the results.

Authority

Authority, defined as, “an accepted source of expert information or advice (<http://dictionary.reference.com/browse/authority>) has the following synonyms: bible, jurisdiction, supremacy, say-so, and strong arm. The word *authority* is derived from French, *autorite*, from Latin *auctoritatem*, meaning “book or quotation that settles and argument” and “invention, advice, opinion, influence, command” (<http://thesaurus.reference.com/browse/authority>). Throughout the data, the idea of who offered advice, opinions and influence in class was mentioned consistently. The nature of the authority theme speaks to the question of who invented ideas and questions, who offered and gave opinions and advice, and whose influence was important in the class. A participant shares thoughts in a reflection about the nature of authority in a class:

I would hypothesize that the students in this class had rarely had the opportunity to ask their own questions about mathematics, let alone raise questions about the validity of formal mathematics [prior to the Foundations of Mathematics Course]. However, what this class enabled us to do was to develop a personal sense of authority in studying mathematics. As our personal authority developed so do our efficacy in analyzing mathematical arguments and judging their validity in reference to our experiences and beliefs. Over the semester we each to varying degrees developed a sense of not only that our questions mattered, but they were important to consider mathematically. Moreover, we learned to listen to our own intuition and, when this contradicted what was mathematically accepted, to delve deeper into both our own understanding as well as the mathematical argument.

In particular I found it beneficial to understand that I was able to decide whether or not a mathematical argument was valid and/or satisfying. Even when I could follow a proof or logical argument, it was still appropriate for me to say, “Sure, I follow that, but it does not effectively account for an answer this question I still have. So, it is not particularly useful for me, and until an argument effectively deals with these, I may follow the argument, but I will not say it is the right argument.”

As the course progressed, there was a greater recognition that we as students of mathematics did not have to “buy in” to an argument that did not make sense personally. Somehow an argument had to be personally satisfying and if it did

contradict particular beliefs or experiences, delving into the argument had to help develop new experiences and intuitions. Arguments had to provide understanding and answer questions. (FM Student Reflection).

In comparison, another doctoral student described the following about a course in which the professor controlled the authority about what was learned and how it was to be learned:

*I took another math course, which was called Dynamics and Chaos. I was introduced to things like countability, fractals, and chaotic systems. The problem sets were really hard. The book was hard to read and he didn't lecture about the book. Like, if you didn't come to class with questions about what you read, he didn't talk about it. He talked about whatever **he** wanted to talk about. It was meant to be you learn it on your own, come with questions, if you don't come with questions, I'm going to talk about whatever **I** want to talk about. (Individual Conversation)*

The theme of authority in a mathematics classroom is more than acknowledgement of an authoritative figure, control, or power. It also speaks to the classroom environment of community, the nature of knowledge, and the manner of classroom discourse. Wilcox, Schram, Lappan & Lanier (1991) argue that teachers among others, need to create a classroom environment of community of learners in order increase mathematical learning. Cobb (1989) agrees by saying:

If we are serious about encouraging students to be mathematical meaning-makers, we should see the teacher and the students as constituting an intellectual community. The classroom setting should be designed as much as possible to allow students to do their own truth-making.

According to John, another student in the class, this sharing in an intellectual community and opportunity to engage in truth-making was “liberating.”

I think it is liberating to me because it was a class about my ideas. It was not a class about somebody else's ideas. Even in the first part where the things we were thinking about were dictated by the instructors and how we made sense of that was up to us, that's my impression... So, liberating in the sense that as a learner, what I'm trying to do is think about my ideas not anyone else's ideas- I mean I'm worried about other student's ideas but not other ideas like privileged ideas. Other

students' ideas are not privileged ideas like those of the teacher. So, the class was about, could we make sense of what our ideas were? (FM Student Reflection)

Interaction

Applebee (1996) reminds us that we “learn to do new things by doing them with others. . . Tomorrow we do on our own what today we do in the company of others” (p. 108). It is the doing together that is embodied in the *interaction* theme. *Interaction* is defined as the mutual or reciprocal action or influence (*Merriam-Webster's Medical Dictionary*. <http://dictionary.reference.com/browse/interaction>). The etymological roots of *inter* and *action* help define the theme. *Inter* meaning “among, between” and *action* from the stem of *agree* meaning, “to do” (*Online Etymology Dictionary*. <http://dictionary.reference.com/browse/interaction>). In other words, *interaction* means, “to do together”.

With the notion of interaction, or “doing together” come ideas of collaboration, community, discourse, being and doing, sharing and being shared to. The “doing together” within a mathematics class can range from a scene of little interaction to a scene where teacher and student roles blur as both parties work together to make meaning. The “doing together” can refer to the interaction between teacher and student, between students, or between the students and the mathematics. Here a student shares a reflection of his descriptions about being interactive in a certain way in mathematics class:

In some ways, my desire to “go to the board” (which in my mind is more of a general phrase for any sort of open-ended public sharing) co-existed with and was co-dependent with my excitement for the course, and ultimately mathematics in general.

Why share? I think of the times I have gone home and shared (what I thought were) interesting mathematical musings with my wife, to be greeted with supportive, though not always interested, nods. In a similar vein, my brother, the writer, is often sought out at family gatherings to get his take on the latest best-seller or Oscar prediction buzz. I don't recall ever being cornered at Thanksgiving to give my 2 cents on the future of two-column proofs in the high-school curriculum.

Please do not misunderstand, I do not offer such statements as complaints. I would neither trade my family for a more mathematical-minded bunch nor my experiences in mathematics for more mainstream interests. But what this course offered, more than Thanksgiving dinner at my mom's cousins' house, was the opportunity to truly share. So, I suppose, more than anything, my going to the board was simply taking advantage of this opportunity to both share and be "shared to" (FM Student Reflection).

According to Hultgren (1995), to let learn means: "To prepare a space for listening that intertwines identities (self/other and self/society) in a retrieval of being, a leading in itself that withdraws from teacher to being-in-teaching together (p. 377).

A student reflects on a space where the interaction was key:

In the space of our class, the students were allowed and required to become listeners. We listened to each other's ideas. Through our listening we came to know each other's thinking. We came to know that [a FM student] always had a unique way of thinking; [another FM student] would always construct a representation to explain; [another FM student] would make connections to high school teaching, and Anne Marie would want to know why (FM Student Reflection).

In high interaction classrooms, students' and teachers' voices are both heard. In some instance, high interaction classrooms the teacher's voices are the once heard least.

Tony Brown describes an episode of high interaction teaching by listening:

*Sitting in on this lesson as an experienced mathematics teacher, I couldn't help but be amazed. As I listened, I heard references to addition, subtraction, multiplication, and equivalence concepts'— all at a level of complexity far surpassing the recommended laid out in the mandated curriculum documents. But what was more surprising was that Tom [the teacher] did not seem to be saying much at all. He wasn't teaching by *telling*, rather he seemed to be teaching by *listening* (Brown, 1997).*

Recent research indicates the importance of mathematics classrooms in which students discuss their mathematics and explain and justify their solutions (Yackel, 2001). Maverich (1999) suggests that students learn through opportunities to explain, justify, and listen to the ideas of classmates. In addition, Krummheuer (1995) indicates that participation in negotiations and argumentation has a positive influence on student's conceptual development in mathematics.

Learning with understanding is a product of interactions over time with teachers and other students in a classroom environment that encourages and values exploration of problem situations, modeling, and argumentation.

In addition to an high interaction community of learners, “establishing a classroom where arguments are made to support conjectures, and where the criterion for what makes sense is determined by students and teacher working together is likely to engender in students a very different view of mathematics from the typical rule-and-procedure orientation (Wilcox et al, 1991). Creating a community of learners with shared responsibility for learning holds the promise of providing such an environment (NCTM, 1989).

Mathematics

Mathematics as a discipline has grown out of centuries of asking questions, probing, going down wrong paths exploration invention frustration. We must forget the history of the discipline when we let children “do” mathematics (Brown, 1997).

It is nothing short of a miracle that modern methods of instruction have not yet entirely strangled the holy curiosity of inquiry. (Einstein, Albert (1879-1955)
In H. Eves *Return to Mathematical Circles*, Boston: Prindle, Weber and Schmidt, 1988.)

Mathematics is the study of the measurement, properties, and relationships of quantities and sets, using numbers and symbols (<http://dictionary.reference.com/browse/mathematics>). Its etymological roots are from the Greek *mathematike tekhnē* meaning “mathematical science and from *mathema* meaning “science, knowledge, mathematical knowledge,” related to *manthanein* “to learn,” from the base *mn*, *men*, and *mon* meaning “to think, have one’s mind aroused”. The mathematical content, how it was discussed, and whose mathematical ideas and questions were important, define the *mathematics* theme.

A student adds her own perspective on the mathematics that was learned in the FM course:

What seems so central here is that the mathematics we studied mattered to us. Changing roles from teacher to student and from question answerer to question poser was sometimes a scary transition, but this transformation had the potential to awaken the mathematical mind and to allow mathematics to be intrinsically meaningful (FM Student Reflection).

Throughout conversations with participants, recollections of specific details of content that was learned in their mathematics courses varied. Participants were able to mention big ideas that were discussed, or mentioned a particular topic that was of interest to them. A student shared:

I took biostatistics which was a statistics course focusing on how to use statistics in medicine and medical research which really didn’t turn out to be all that different from another statistics course except we talked about medical studies. It was a lot of lecture and we looked at situations. You could bring in a sheet with formulas for the exams and I did great- I got an A plus great. I didn’t learn anything more than any other statistics class (Stage I Conversation).

In thinking about *mathematics* as a theme, I was drawn to the contrast in descriptions of the mathematics that students engaged in during many of their classes, such as the aforementioned statistics courses, with descriptions of the mathematics that was explored in the FM class. The following mathematical descriptions were gleaned from FM class journals.

Student 1:

Today's class was just great! This idea of the difference between counting and measuring is taking so much shape. I had never thought about it before! It seems incredible that I have finished a whole degree in mathematics and had never thought about it. This is one of those many (probably infinite) things you "know" but you have never really thought about it, questioned it, explicitly used it, and therefore, deeply understood it.

Student 2:

Measuring is about comparing things. One way to get at it is by counting. But sometimes you aren't able to compare by counting (e.g. counting 2 different infinite sets). And in measuring you can. e.g.:



So what is measuring then? Not necessarily counting but comparing to some standard. Don't you compare to some standard when you count? Aren't you comparing to the standard of zero (e.g. how many more_ are you away from zero on the line?)?

Student 3:

"Measuring" is a very loaded word. It strikes me that there is a way (I think) to define it so that it encompasses counting...sort of in the way [the instructor] mentioned about his algebra classes. I side with [FM student] in making a pragmatic argument → You go with a method (or perspective) that results in a solution and toss the one that doesn't. However, I am intrigued why these different methods work in some cases and not in others. I'm willing to chalk it up to the mysteries of infinity and the competing metaphors that [visiting Post Doctoral Fellow] spoke about. Anne Marie seemed to express some skepticism that I should be able to "continue the pattern" into infinity which reminds me of the assumption that Brian Rotman points out that we all make. What if really large numbers don't behave the same as their more accessible counterparts?

Student 4:

“Counting” means comparing the elements in sets, the number of elements, comparing the number of elements. The methods labeled “measuring” is more difficult to pin down. In two of the cases we’ve been talking about it involves area and length, and for these I have a sense of what it means to measure. Define a concept, chose some standard unit, then describe some quantity of the concept in terms of the number of standard units. [gosh, that sounds a little like counting!]. . . . I like my sense that those ideas are a kind of “density” argument. That might be a way to reconcile, or find a more general statement of what is meant by measurement. As it stands those ideas get lumped into measurement because they get me the “right” answer.

Student 5:

Wow! Today’s class was exactly my issue with math. When do we think in one way in some situations and in other ways in other situations? Particularly, when do we think in “intuitive” way and when not? It’s interesting, though, that in the times we don’t think according to intuition, we try to make it intuitive! ☺

Sense of Purpose

Constructivists must attend to the fact that purposes are constructed as well as knowledge, and students have a wide variety of purposes. Just as we need to know how students think if we are to help them build powerful mathematical constructions, we need to know them as persons if we are to assist their construction of well chosen purposes. (Noddings, 1993, p. 159).

Purpose, according to *Dictionary.com Unabridged*

(<http://dictionary.reference.com/browse/purpose>) is the reason for which something exists or is done, made, used. The theme of purpose uncovers the intended or desired result, goals and makes clear the point of particular mathematical experiences. The word *purpose*, come from O.Fr. *porpos* "aim, intention", from *porposer* "to put forth," from *por-* "forth" (from L. *pro-* "forth") + O.Fr. *poser* "to put, place" (<http://dictionary.reference.com/browse/purpose>).

"A sense of purpose and future signifies goal direction, educational aspirations, achievement motivation, persistence, hopefulness, optimism, and spiritual connectedness." (Bernard, 1995, p. 67) The theme of purpose speaks to the following questions: What were the goals of taking a class? What was the nature of the motivation

in doing the mathematics? What was it that gave students optimism in their mathematics learning and pushed them to ask questions? What were students' intentions in doing the mathematics?

The participants' notions of purpose were related to the purpose of taking the class, purpose of the content, and/or the purpose of engaging with the mathematics. Richards (1991) describes purpose in mathematic classes as being either "reason- giving" or "reason- seeking". This has provided a useful way of thinking about the purpose of doing of the mathematics.

Design of Data Chapters

The range of courses taken among the pool of participants varied greatly. Within their doctoral programs, the courses fit into three categories: courses labeled below the 200 level within the mathematics department, courses above the 400 (but below 600) level within the mathematics department and courses labeled 700 level mathematics courses offered in the education department specifically for mathematics education doctoral students. According to the online University course catalog, 200 level courses are primarily offered to undergraduate sophomores, 400 level courses are intended for undergraduate upper-class students and/or graduate students, and 700 level courses are limited to graduate students only.

Structure of chapters

I engaged in conversations and data analysis with each participant surrounding each category of classes that was relevant to their mathematical coursework. The two chapters report on course experiences that are classified as either *typical* or *novel*. The data in the chapters are shared in two stages. First, I report participants' descriptions of

the experience- either *typical* or *novel* mathematical course experiences. These initial descriptions are presented through the use of the phenomenological tool of “bracketing” (van Manen, 1990). Bracketing is an attempt to “suspend one’s various beliefs” (van Manen, 1990, p. 175). The method of “bracketing” lifts an item under investigation from its meaning context with all judgments suspended (van Manen, 1990). The bracketed course experiences are not evaluated as being either true or false, good or bad. These descriptions, as reported by participants, address research questions. The purpose of the presentation of bracketed experiences is to provide the reader with a *descriptive* account, as free of judgment and personal analysis as possible, of the essential features of the courses. The presentation of data in this manner provides insight into the following research question in this study: What is the nature of the mathematical course experience that mathematics education doctoral students identify as components of their program?

The meaning or essence of a phenomenon is never simple or one-dimensional (van Manen, 1990, pg. 78). Therefore, a second level of course experience descriptions is provided. The purpose of these descriptions is to attempt to present the human meanings of the experiences. These meanings are cultivated by participant thoughts, emotions, points of view, and subjective realities. In these descriptions, referred to as “Experiential descriptions”, participants share their emotions, perspectives, preferences, expectations, and feelings about their experiences. These provide a description of the essence of the course experiences and answer the questions of “What was it like to be a student in a course?” “How did these experiences make participants feel?” “What affect did the experience have on participants?”. The presentation of data in this manor provides insight into the following research questions in this study: What is the meaning of these

experiences in the context of doctoral student careers? How are these experiences situated within the preparedness as mathematics education doctoral students enter the mathematics education field? How can specific mathematical experiences influence the identity and preparation of a mathematics education doctoral student?

In each of the data chapters, course experiences are shared through each of the four themes: authority, interaction, mathematics and purpose. First, a brief introduction to the theme is provided. These introductions pay particular respect to the course experience type (*typical* or *novel*), and build upon the general theme descriptions provided in chapter four. Then, bracketed features of the course experiences with respect to the theme are provided by several of the participants. Next, one selected participants' voice highlights the theme through experiential descriptions. Individual participants were selected by the researcher based on several factors including but not limited to the level and saturation of a participant's voice on a particular theme. As described earlier, particular themes appeared more often and with more intensity and emotional for some participants than others. The presentation of data is repeated for each of the themes. Lastly, an analysis and conclusion are provided.

Mathematical Interlude Two is provided next. Following the interlude is Chapter Five which reports on participants' descriptions of their mathematical experiences in courses classified as *typical* by the researcher.

MATHEMATICAL INTERLUDE TWO

The infinite! No other question has ever moved so profoundly the spirit of man.
Hilbert, David (1862-1943) In J. R. Newman (ed.) *The World of Mathematics*, New
York: Simon and Schuster, 1956.

I distinctly remember the day that I first heard the news. It was one of those memorable days in your life that forever changes how you think about the things ahead of you and really makes you wonder about the life you've been living up to that moment. For me, it caused great angst. I felt partly betrayed, very confused, and at times angry about the entire situation. How could all of these teachers been hiding this from me? I had gotten to a place intellectually where I couldn't just accept the news without some serious debate. I had been in the business of student disequilibrium and mathematical argumentation for some years now. But what was I to do now? I was surrounded by people whom quite clearly had known this for years and had moved on with their lives. To make them stop now and ask them to explain and defend their beliefs seemed unnecessary. I suspect that if I had heard this nonsense in any other class or at any other time in my life, I might accept it and move on, but not now.

Children learn counting at a very early age. Parents, many unknowingly are helping children develop one-to-one correspondence while encouraging them to count their Cheerios as youngsters begin cutting teeth. From this initial teeth cutting experience well into elementary school, students practice the beginning of the natural numbers 1,2,3... The sequence of numbers gets bigger and bigger as children grow older. I don't imagine regular discussions occurring with students this young around the nature of infinity but I know that they develop a sense of "onemoreness" early on. Listening to my three year old daughter discussing how many bugs might be crawling around the park

shows her understanding of infinity, although immature, as always being able to add one more:

Natalija: “There's like 400 hundred red bugs.”

Micah: “No- like a million!”

Natalija: “Maybe a million and one.

Numbers between 1 and 2 naturally become part of children's' world as they find the need to start sharing. For a while, "half" exists as a verb rather than a number existing midway between 1 and 2. Then somewhere in the middle of elementary school, a teacher breaks the news that there are a bunch of special numbers (not just “half”) that we somehow skipped over for all of these years. This coincidently is where we lose many prospective mathematicians. Fractions are these funny numbers that don't play by the same rules that we have spent years perfecting since we started school. They don't act like the other numbers. They don't sound like other numbers and they don't look like other numbers. And there are so many of them. But in the beginning, we're eased in slowly and don't find out about all of them. We spend most of our time with the biggies: $1/4$, $1/3$, $1/2$, $2/3$, $3/4$. Little do we know at the time, but it would be impossible to ever learn all of them. In fact, we will never encounter most of them. Some of us “get it” and some don't. And of those that “get it”, I wonder how many people really get fractions in the same sense that most of us get $1, 2, 3, \dots$. I happened to be a student who got it- at least for a while.

As we continue through elementary and middle school, we are introduced to the rest of the numbers (does this suggest that the others are less real?). We are presented with a myriad of terms to described particular categories of numbers: Natural, Counting,

Whole, Terminating, Repeating, Non-repeating, etc. And in all these cases, we discover some relative of the million and one bugs. That is, the idea that there are some numbers that never end. How is that possible? Everything ends. All the numbers I've learned about have an end. It's where I was taught to go to in order to start multi-digit multiplication. How would I do that if a number didn't have an end?

Then, much later, my story is again interrupted by some shocking news- One of these non-terminating decimals, a number with no end, is considered to be equal to one of the most basic numbers- a whole number. How could .9 repeating being considered equal to 1? It systematically goes against all that I know about number. In class, we have been struggling with issues of infinity and the lengths and measures of the numbers between 0 and 1. I am starting to feel less uncomfortable and more curious. Are these new ideas about the Real numbers that I'm learning helping me to understand why .9 repeating and 1 might be equal or are they part of the brainwashing that exists to get us to buy the argument even though it goes against our guts?

The topics, such as the .9 repeating incident, that arose in our Foundations of Mathematics class were untypical discussions, in nature and content, for a college-level mathematics class as they were often driven by student ideas and questions. The notion that .9 repeating is equal to 1 emerged in one class session. At the time, no one could have guessed that this idea would cause so much discussion, debate and consideration as it did. I know I was immediately taken aback having never heard this mathematical idea. Was someone suggesting that .9 repeating was really the same exact number as 1 or was it some idea pushed by reality that the .9 repeating acts like 1 because its so close to it

and for practicality, we could just round up. If in real life I owed someone $.99$, I would simply give them \$1.

My classmates, most of whom had much more formal mathematics education than I, were convinced of their equality. Many times I felt as if they would bring it up and offer some mathematical nuggets to try and convince me to see it their way.

Unconvinced, I felt it was ridiculous and felt safe enough to speak out in class. In a way, I felt as if I was up against a wall since most of my classmates believed the idea. There was a part of me that was somewhat reluctant to speak out because it might publicly expose the holes in my own mathematical thinking. But on the other hand, this idea seemed inherently wrong. I could not accept that two distinct numbers could be the same.

After much debate, the discussion of the limit of $.9$ repeating was brought out. I suddenly thought about this statement as being possible- that the limit of $.9$ repeating may be equal to 1. The limit, or boundary of the sequence is 1. It will never reach one exactly but it will get so close to 1 that it could act like 1. But the number $.9$ repeating cannot be equal to 1. For 1 and $.9$ repeating have different places. The limit of $.9$ repeating has a space. It is 1. But we cannot say that these two numbers have the same place.

As it came to be, the $.9$ repeating = 1 idea did not continue to be a specific problem to be solved or decided upon. Rather, it became a catalyst for a deep involvement and consideration of our exploration of real numbers. The exploration provided us a space for learning, conjecturing, debating, and sharing. It was a safe place. It was a safe mathematical environment to question, wrestle, and explore. It was even a safe place to not-know.

CHAPTER FIVE

Introduction

This study seeks to understand mathematics education doctoral students' mathematical experiences within their doctoral preparation. Throughout conversations and communication with participants, it was evident that the experiences of participants fell along two distinct lines: *typical* and *novel*. This chapter reports on participants' descriptions of their mathematical experiences in courses classified as *typical* by the researcher. In participant's descriptions of their *typical* experiences, I found similarities in the descriptions along the dimensions of authority, interaction, mathematics and purpose.

As described in chapter four, the data in this chapter are shared on two levels. First, I report participants' descriptions of *typical* mathematical course experiences. These initial descriptions are presented through the use of the phenomenological tool of "bracketing" (van Manen, 1990). The presentation of data in this manner provides insight into the following research question in this study: What is the nature of the mathematical course experience that mathematics education doctoral students identify as components of their program?

Then, a second level of course experience descriptions is provided. The purpose of these descriptions is to attempt to present the human meanings of the experiences. In these descriptions, referred to as "Experiential descriptions", participants share their emotions, perspectives, preferences, expectations, and feelings about their experiences. The presentation of data in this manner provides insight into the following research questions in this study: What is the meaning of these experiences in the context of

doctoral student careers? How are these experiences situated within the preparedness as mathematics education doctoral students enter the mathematics education field? How can specific mathematical experiences influence the identity and preparation of a mathematics education doctoral student?

In this chapter, course experiences are shared through each of the four themes: authority, interaction, mathematics and purpose. First, a brief introduction to the theme is provided. These introductions pay particular respect to *typical* courses, and build upon the general theme descriptions provided in chapter four. Then, bracketed features of *typical* course experiences with respect to the theme are provided by several of the participants. Next, one selected participants' voice highlights the theme through experiential descriptions. Individual participants were selected by the researcher based on several factors including but not limited to the level and saturation of a participant's voice on a particular theme. The presentation of data is repeated for each of the themes. Lastly, an analysis and conclusion are provided.

As previously mentioned, only two participants in this study were required to take mathematics courses below the 400 level. This set of courses included Calculus I and Calculus II. All six participants in the study were required as a part of the doctoral program to take 400 level mathematics courses taught within the mathematics department. The 200 level courses and the majority of the 400 level courses taken by participants emerged in descriptions of courses that provided experiences identified as *typical*. The experiences of participants in 200 level Calculus course experiences and several 400 level mathematics courses are contained in this chapter.

Typical Mathematical Course Experiences

During conversations conducted early in data collection, the following chunks of descriptive language of participants' mathematical course experiences within their mathematics education doctoral program emerged:

- *Professor talking to the board.*
- *Lecture.*
- *Follow the steps.*
- *He was not able to explain the math.*
- *They didn't know how to teach.*
- *We didn't ask questions.*
- *There were no projects.*
- *We didn't work together.*
- *There were a lot of problems.*
- *There were a lot of handouts.*
- *They knew "it" but they couldn't teach "it".*
- *Textbooks.*
- *Proofs.*
- *Bringing sheets of formulas into the final.*
- *Professors talking about what he wanted to talk about.*
- *We worked individually.*
- *Blah blah blah.*

By continuing to engage with participants in individual and group conversations and probing more deeply about such descriptions of their course experiences, I found similarities in the descriptions along the dimensions of authority, interaction, mathematics and purpose in *typical* courses. It is these mathematical experiences and their affect on participants, all future Stewards of the Field of mathematics education, that are unpacked in this chapter.

Authority

Throughout the data, the idea of who offered advice, opinions and influence in class was mentioned consistently. The nature of the authority theme speaks to the question of who invented ideas and questions, who offered and gave opinions and advice,

and whose influence was important in the class. All the participants spoke to the nature of authority in *typical* mathematics course situations as belong to the instructor and provided images of traditional mathematics classrooms.

Bracketed experiences about authority.

Wendy's description about her perception of the authority in the 200 level courses (2 semester Calculus series) was that it, "belonged to the professor." She shared that the assumed authority seemed to be felt by students and was reflected in the nature of the course environment. "Whatever he said was gold, students merely took notes, and copied every word he uttered. No one ever questioned him." Although Wendy described the authority as belonging to the professor, she acknowledged ways that students may have revolted against or coped with their lack of voice. "Students may have usurped his authority by, "not paying attention-either reading the paper or sleeping in class."

In Mia's reflections on her experiences in the 400 level courses that she took, she did not provide a single example where the instructor was not perceived to be the sole authority figure in the class. In most of these courses, the instructor, "lectured for the entire class period." This idea of authority also permeated Mia's descriptions of the decision making about what was to be learned, and how and when it was to be presented. She described the role of the student in these courses as receiver of the knowledge that the instructor provided. In these classes, Mia recalled that students generally did not talk to the professor or to each other during class. To Mia, this lack of student voice was confirmation that the authority of the experience belonged to the instructor.

In the majority of Faith's reflections about her 400 level mathematics courses, she also suggested that the professors who taught the courses held the authority in the class.

She indicated that the professors determined what was to be learned, in what way, and by what pace. She furthered the notion that, “the professor was the authority over many facets of the course experiences including the knowledge to be learned and that he referred to a textbook. That is, the professor had the knowledge to be learned, told or explained knowledge to us students, required us to complete tasks in the textbook, and we were expected to demonstrate our understanding on what was presented as measured on assessments (homework, quizzes, tests).”

Kristen shared that typically the, “professor and the textbook were the sole sources of authority” in the course and that students’ voices were minimal in class sessions. By the time Kristen entered the doctoral program, she had already taken most of the 400 level courses available to her. The overwhelming majority of these courses, according to Kristen, were taught in lecture style. They were, “traditional...professor was at the board, putting things on the board and we were trying to copy as fast as possible. I would occasionally ask questions but there weren’t many questions. Problems sets for homework every class so, then the next 24 hours were spent working on the problems.” Kristen shared not having thought much about her experiences in these classes until she was afforded a very different mathematical course experience later in her program of study.

Although the majority of mathematical course experiences I was afforded in my doctoral program are most closely aligned with the *novel* course experiences in this study, my calculus course experiences are most closely aligned with those described as *typical*. The calculus class was fairly characteristic of what one might imagine a large university mathematics course would be. The lecture room was very large and the seats were bolted

to the floor in long rows. Students were required to sit while the professor had the freedom to move around. On most days, the seats were filled with students before the professor came through the door with his book in hand. The professor would enter the room and started writing on the board immediately while students started copying. The professor was in control of the mathematics that we discussed and ‘learned.’ His authority was even in the pages of the textbook as he was a co-author.

Faith on authority: Experiential descriptions.

Faith reported that her mathematical course experiences were mostly frustrating and disappointing. Specifically, Faith shared that, “my ideology and the ideology of the [doctoral program] seemed for the most part aligned. The inclusion of course work in traditional mathematics classes, with traditional pedagogies, did not resonate with these ideologies (mine or the programs).” This, according to Faith, caused a disconnect, or contradiction in her learning experiences.

I get stuck in this abstract algebra course. I wouldn’t say stuck but okay, I take this abstract algebra course which is this 400 level undergrad math course and that was probably the most humbling experience I’ve ever had as a learner of mathematics. I had no idea what anybody was talking about (Faith, reflection).

Faith spoke often of the professor’s role in her 400 level courses. According to Faith’s perception, the instructor was the sole authority figure in the class most of the time. She characterized the typical courses as mostly taught using traditional lecture-style methods with minimal-to-no connections to school mathematics with the exception of two course experiences. Faith described her interaction experiences in an Abstract Algebra course as “awful”.

The majority of Faith’s descriptions indicated that she felt the professor determined what was to be learned, in what way, and by what pace. In addition, Faith

acknowledged feeling the professor's authority in the assessment and grading systems in the courses.

Faith's memories of these experiences were distinctly painful as a person who had experienced mathematics in a very positive way up until graduate school. She described the mathematics as hard and shared the following memories:

I only really began to understand the mathematics in the course material when I worked individually with [the mathematics education faculty member]. I have never experienced mathematics the way I had when I got to grad school. Having done well throughout K-12 schooling in mathematics, my transition into undergraduate mathematics seemed rather seamless. I grew up loving mathematics, I was enrolled in a science and technology high school, and I went on [to the university] as an engineering student so I really didn't consider mathematics to be an obstacle for my success. It wasn't easy for me, but compared to most, I seemed to have a knack for it. ... Fast forward to Fall 1999 (my first semester in grad school), when I attempted to take an abstract algebra course by the advisement of members of the mathematics faculty, I remember the first day of classes thinking, 'What the hell?' I had no idea what in the world the professor was talking about and then worse off I had no idea what he was writing on the board. It all seemed so foreign. I felt ill. The semester got no better and after a handful of failed quizzes and tests, I opted to drop the course, with my confidence in mathematics totally bruised and shattered. I think I used to describe that semester as "traumatic" for me in terms of my mathematics learning (Faith, reflection).

Faith's experience in the abstract algebra course was one that surprised her as she had grown accustomed to succeeding in mathematics and overall had been an excellent student.

She was also concerned by the irony that existed in her learning of mathematics education with the experiences of learning of mathematics that she had in the mathematics department.

It seemed as though taking mathematics courses were part of an unsaid hazing process where one had to prove oneself worthy of earning a degree in mathematics education...I recall something a faculty member said to me as I questioned the purpose of having to take these courses. The comment went

something like this, “How could you earn a degree in mathematics education and not *know* any mathematics?” Ironically, what it means to *know* mathematics in the unit (mathematics education) in relation to earning a graduate degree in mathematics education, seems to be heavily defined by the number of upper-level mathematics courses one takes and one’s success in these classes (Faith, reflection).

It was Faith’s perception that the faculty had interest in taking more mathematics but Faith was adamant that majority of her course experiences were “negative” rather than complementary to her education coursework. Faith’s reflections of being a learner in the course reflect the importance of how the mathematical learning experiences become an element of what one is studying in a mathematics education doctoral program. Faith appeared to be aware of this dichotomy and angered by the disconnect in the midst of her mathematical experiences.

Being a mathematics education graduate student, focusing on the teaching and learning of mathematics, is an important distinction from other non-mathematics education students in the course. Mathematics education graduate students have a wide range of professional goals that are directly related to the discipline. This fact requires mathematics education doctoral students to know mathematics in particular ways. The goals and implications for taking a mathematics course, then, are different for a pre-med graduate student than it is for a mathematics education graduate student.

Faith did have two experiences that proved to be exceptionally positive. In both of these classes, the instructors’ roles were much more collaborative. These experiences are presented in more detail in the next chapter, which focuses on those experiences reported as *novel*.

Interaction

College teaching and lecturing have been so long associated that when one pictures a college professor in a classroom, he almost inevitably pictures him as lecturing (McKeahcie, 1963, p. 1125).

A second theme that emerged from discussions with participants on their mathematical experiences was related to the notion of interaction. The interaction in class, or “doing together” referred to ideas of collaboration, community, discourse, being and doing, sharing and being shared to. The “doing together” within a mathematics class can range from a scene of little interaction to a scene where teacher and student roles blur as both parties work together to make meaning. The “doing together” can refer to the interaction between different classroom relationships: between teacher and student, among students, or between the students and the mathematics. In course experiences deemed “typical”, the “doing together” was consistently described as consisting of little to no interaction in each of those relationships.

In these *typical courses*, the interaction between teacher and student was consistently described as limited. The interaction that occurred was primarily one-way. Participants described students in these typical courses almost never initiating an interaction except at designated question and answer sessions. Students primarily took notes while the professor lectured. Participants reported little to no interaction among students in lecture or discussion sessions. The instructor-student interaction consisted primarily of questions being asked at the end of the class. The interaction in class was even less between students and the interaction between the professor and students as “students raising their hands to ask questions for clarification purposes (but this didn’t happen often)”. With regard to the interaction with the mathematics, participants

provided few examples of students interacting with the mathematics outside of limited interaction that mainly consisted of completing problem sets for the class and studying for quizzes and tests.

Bracketed experiences about interaction.

Reflecting on 200 level course experiences, Wendy described the interaction between teacher and student as limited. The interaction that occurred was primarily one-way and, “limited to him talking and students trying to capture his every word in their notes.” Students almost never initiated any interaction except at designated question and answer sessions. She reported there being little to no interaction between students in lecture or discussion sessions.

In addition to the lack of student voice in the 400 level courses, Mia reported there was virtually no interaction in her classes. “Students entered the room, sat facing front in individual desks, listened to the lecture given with PowerPoint slides, and filed out when the instructor said, ‘And that’s where we’ll stop today’.” Mia’s recollection of her experiences in 400 level courses was that they were consistently non-interactive. Students primarily took notes while the professor lectured. Interaction was minimal between student and instructor. The instructor-student interaction consisted primarily of questions being asked at the end of the class. The interaction in class was even less between students. Mia reflected that there was “no interaction between students – no conversations, no eye contact, no greetings.”

Mia shared that there were no group or partner projects or interaction regardless of the ripe opportunities for students to work together with class sizes at the 400 level courses being relatively small. Students did get together and do homework outside of

class. Through these homework and study session, Mia reported some bonding with other students. These bonds, however, were neither made, nor sustained in the mathematics class itself.

Faith's general characterization of the interaction between students in her 400 level classes was "practically non-existent" and the interaction between the professor and students as "students raising their hands to ask questions for clarification purposes - but this didn't happen often." With regard to the interaction with the mathematics, Faith reported limited interaction that mainly consisted of completing problem sets for the class and studying for quizzes and tests. She reported spending many hours in tutoring outside of her courses with a mathematic education faculty member who spent time working with her to make sure she, "was making sense of the mathematics. In these sessions, I was expected to answer the questions he raised as a means to gain understanding of the concepts that were presented in class. There was certainly a sense of accomplishment when something 'clicked' and I got something."

Eva commented on being used to mathematics course as having "minimal" interaction if any. "the teacher lectured or answered questions" Eva shared that in classes, she, "never asked a question- never felt comfortable doing so. People would ask questions I did not understand, so I just kept my mouth shut so as not to come across as stupid."

When thinking about the calculus lectures, I don't recall much academic or mathematical interaction during the calculus lectures. Students sat in rows and copied notes from the board for an hour. There was a set time at the end of lecture for students to interact with the professor through a question and answer session and that many lectures

would end without a single question from the audience. If any, the questions were specifically concerned with a problem from the homework that someone just couldn't get right (as dictated by the answer in the back of the textbook). The class would end almost as abruptly as it would begin and students filed out of the room with a sigh of relief.

Anne Marie on interaction: Experiential descriptions.

The 200 level calculus series in my doctoral program seemed to typically describe a large college mathematics course. I was disappointed in having to taking a series of classes that I had successfully taken in the past, both in high school and undergraduate studies. However, as part of my acceptance into the doctoral program, I was required to take the Calculus serious. It had been several years since I had taken any mathematics courses and it was intended to be a way for me to prepare for taking the 400 level mathematics courses in my program of study.

There was almost no academic or mathematical interaction during the lectures. Students sat in rows and copied notes from the board for an hour. There was a set time at the end of lecture for students to interact with the professor through a question and answer session. Many lectures would end without a single question from the audience. If any, the questions were specifically concerned with a problem from the homework that someone just couldn't get right (as dictated by the answer in the back of the textbook). The class would end almost as abruptly as it would begin and students filed out of the room.

The interaction in the 200 level courses was extremely limited and occurred primarily in a "one-way" fashion with the instructors speaking to the students. Students were not engaged in a way that required interaction with the instructor (with the

exception of asking questions during a designated time) or with each other. From my perspective, there appeared to be little interaction between students and the mathematics outside of practicing and memorizing what was given in class.

It was my perception that the purpose of the doing of mathematics in the classes was to learn or memorize the mathematics that someone else had once done. This was shown in the ways the students were asked to engage with the mathematics- by copying and knowing what the professor said and by practicing and memorizing theorems that were worked out in lecture.

Dewey suggests that traditional approaches to education rely on bodies of information and of skills that have been worked out in the past; therefore, the chief business of the school is to transmit them to the new generation” (1938, p. 17). This is in accord with my reflection of the purpose of the class as being to do the mathematics that someone else had once did in ways that required little interactions with other learners or the professor.

My reactions to the mathematical experiences in the *typical* courses are shared in the following journal excerpt. The journal was an artifact from a phenomenology course that I took during my doctoral program. I was enrolled in the course at a pivotal moment in my doctoral career. It was in this semester that came to the dissertation topic and questions. Encouraged by the instructor, I engaged frequently in personal journal writing about my mathematical course experiences.

I'd like to ask a question, but instead I sit silently in this class they tell me I need to take. They tell me that if I am going to be respected in the field, I certainly need more of this. I couldn't possibly earn a doctorate in mathematics education without taking more mathematics classes. Maybe I can ask after class. After all there are 200 or so students in the lecture hall and I am intimidated by the thing that seems to happen anytime anyone asks a question in the class. And so I

continue to take notes. I've taken years of tedious notes. My homework has been turned in and I've aced all the quizzes that have been given up to this point. I'm bothered as the professor rambles on about the area under a curve that he's attempting to draw on the board. I can't help but wonder about the importance of my presence here. My suppressed resentment for having to sit in this class is encouraging me to raise my hand. But, I don't give in. He continues to lecture about the slope of some line and tells me its connection to something in my life. I'm briefly interested. And the same question keeps rising to the surface of my thinking but I can't find the courage to ask. So, I stay quiet, but not for the purpose that Heidegger (1997/1926) advises, which is to stay quiet in order to let something be understood.

True understanding seems to be quite irrelevant in this space. Understanding implies owning some knowledge, and in most of these mathematics classes students become tenants. As in most owner-tenant relationships, a good tenant is compliant with the owner's rules and is careful not to disturb the owner's property. In mathematics classes, students have traditionally been expected to receive the knowledge the teacher is giving and not to mess up any of the neatly packaged mathematical ideas served in class. Being quiet can be seen as the sign of a good tenant. I've been a good tenant in my mathematics classes for quite some time. I've been quiet for almost as long as I can remember. And always the same question seems to press against my lips but it doesn't seem to escape. And now, as a doctoral student in mathematics education, I've come so far that it seems inappropriate to ask now. I've gone through a master's program focused on mathematics education and am pursuing a lifelong career in this field. So, is it really time for me to ask the one question that continues to persist mathematically in my mind? When can I finally ask, WHY BOTHER? (Personal journal, 2004).

My frustrations are evident in the journal reflections. I was frustrated for having to retake calculus but even more frustrated that the mathematical learning experiences were so clearly disjoint from the kinds of mathematical experiences I was concurrently learning about in my mathematics education courses.

Mathematics

The nature of the mathematics within mathematics courses emerged as a critical theme in the study. According to Rosenthal (1995), advanced mathematics courses are often taught primarily using the lecture format, which promotes passivity and isolation in students. The reports by participants were consistent with Rosenthal's description and

painted a picture of being exposed to mathematics that was heavy with lecture, memorization, textbook work, and “redoing” of mathematics that others had done.

Bracketed experiences about mathematics.

I do remember thinking that this was the first time I could apply the adage ‘Can’t see the forest for the trees’. The trees were the separate formulas we had to memorize and the forest was Calculus itself. Although I could somewhat see the trees (meaning that I could find a derivative or that I could differentiate- back then, certainly not now) I still did not know what Calculus was all about (Wendy, reflection).

Wendy’s reflection on the mathematics that she experienced in the 200 level courses was one of traditional memorizing and formula applying. “The math was a bunch of formulas and trying to remember when to use which.” The content was not at all explicitly important to Wendy in thinking about her own mathematical understanding. She reflected, “I knew how to calculate derivatives or to differentiate, but I could not figure the why.”

Mia distinguished the mathematics that she was learning about in her math classes as “Mathematics with a capital M” from the mathematics she felt she would be using in teaching of students.

The classes that I took, none of them were related to things that I would teach. I guess to the professors, it seemed like pretty elementary mathematics but to me it seemed completely irrelevant. Especially because I had taught quite a few things in the high school mathematics curriculum (Mia, reflection).

Mia mentioned sessions at mathematics education conferences that she had attended that shared information about mathematics courses that would be relevant and meaningful for mathematics teachers and mathematics educators. Mia shared that these classes that looking in depth at and experiencing the mathematics you would be required

to teach, would “appeal more to more to me than, ‘What’s the formula?’ and ‘Plug and Chug’.”

Kristen shared that the mathematics she experienced in many courses “existed before us – there was nothing for us to discover. We were to memorize and apply theorems.” Kristen expounded on this idea in her description of the mathematics she experienced in a non-typical course:

In my experiences in other math classes, I had a sense of how to solve a problem simply by looking at when in the course the question was posed; the proof required only the theorems on the previous pages. Because the teacher assigned the questions, I trusted that I had all the knowledge and skills required to answer the question... In math courses, students often look in the back of the book or other textbooks to find the solution to a question assigned by the instructor.

Eva also commented on the nature of mathematics in many of her courses. For example, when referring to a Dynamics & Chaos course, she said, “The mathematics ideas was actually very interesting, though again, it was pretty much something we learned on our own, and then listened to the professor's research or whatever. Doing mathematics was then a roughly individual experience.”

For me, the mathematics in calculus felt recycled. The mathematics experienced in these lectures was identical to the mathematics I had experienced in other calculus courses in prior calculus course experiences (undergraduate courses).

Although I couldn’t remember exactly what it was that I had learned year’s prior, the mathematics was easy to re-learn. It was concept driven. We learned theorems, rules, principles, and procedures. The professor lectured about them, showed us their relevance and usefulness, and continued with several problems. It was mostly black and white. We practiced the problems and went over homework problems in the weekly discussion.

There was no discovery. One thing was clear- the mathematics was not mine to discover or even think deeply about.

We learned the mathematics in the following way: The professor would present a theorem or rule in calculus (these were presented in order of section location in the book- 1.1, 1.2, 1.3, etc.). He would present its importance, how it was derived, show several examples and then move onto the next section. At the end of class, students could ask questions about what they had seen and heard.

Mia on mathematics: Experiential descriptions.

Mia spoke candidly about her mathematical experiences and of the feeling like not really having opportunities to understand mathematics that was important to her but rather having to take classes that were about “mathematics with a capital M”.

I think what happens is, the more you take, the more you’re building on a foundation that was weak to begin with and so I am kind of standing at the tippy top of that very shakily built foundation (Reflection, Mia).

She expressed the irony in taking mathematics education courses that contradicted the learning she experienced in her mathematics classes:

So I felt it was a real contradiction because I was in my education courses about how to teach [mathematics] while I was taking my math courses. Well none of these professors [mathematics professors] had to take any of these courses [education courses] and it was pretty clear that they don’t either know how to teach or believe that there are other ways to teach other than standing at the board and lecture for an hour or two hours whatever the class time is. You put your chalk down and you leave. And there’s not really this question and answer period (Reflection, Mia).

For Mia, her experiences of 400 level courses solidified her belief that the mathematics that she was responsible teaching and sharing with others, was not the same, or even relevant to the mathematics she was required to take in her program. According to Mia, the 400 level courses she took for her doctoral problem offered the same kinds of

mathematical experience she had during her undergraduate and graduate programs. The taking of these courses was to check off requirements on her program of study.

Mia was clear in her descriptions that the mathematics she encountered in her 400 level courses was not the mathematics she felt she would be using in her future. The 400 level mathematics courses represented Mathematics with a capital M for Mia. Mia describes the mathematics in these courses as challenging, leaving her to feel confused, and having nothing to do with the mathematics she was to teach. Mia confided, “I struggled to make sense of the lectures and the homework assignments. The instructors must have had office hours but I don’t think they were convenient or maybe I just didn’t think that would have been helpful.” Mia shared that, “none of my math experiences in my doctoral program have been meaningful. And I think part of that is the people who have taught it. Their background is not education- it is mathematics. They know their content but they don’t know how to convey that content.”

Purpose

The theme of purpose uncovers the intended or desired result, goals and makes clear the point of particular mathematical experiences. The participants’ notions of purpose were related to the purpose of taking the class, purpose of the content, and/or the purpose of engaging with the mathematics. Richards (1991) describes purpose in mathematic classes as being either “reason- giving” or “reason- seeking”. When reporting on *typical* course experiences, participants seldom referred to a purpose. When questioned about their sense of purpose, participants’ descriptions of the purpose in taking the courses was to check off mathematics course requirements on their program of study. When participants mentioned the purpose of the mathematics in these courses, the

distinction most closely resemble the “reason-giving” description made by Richards (1991). This theme is highlighted as important particularly when contrasted to the detailed and in-depth participant descriptions of the purpose of *novel* courses, described in the next chapter.

Bracketed experiences about purpose.

The theme of purpose, or more specifically lack of purpose, was one that permeated all of Wendy’s reflections on her mathematical experiences in her doctoral program. With respect to Calculus, Wendy expressed, “I couldn’t figure out, and nobody could tell me why, I needed to be there.” Her experiences were ones to “just get through.” She shared, “I managed to get through it and I only got a C the first time around. So, I got a C and then a B.”

Mia’s sense of purpose for having to take more mathematics was to check off the requirements and satisfy the mathematics education faculty. Mia spoke of being “made” to take mathematics classes as a way to “get enough” mathematics for her degree. This sense of needing to “get more” was due in part to the influence of advisor and the doctoral program requirements.

They did make me take another math course in spring of 2004. [My advisor] said I didn’t have enough math. I had to take a combinatorics class. I remember sitting in her office and I had an electives course and I wanted to take a statistics courses and she said, “No. You’ve got too much statistics. I want you to take a math course. So we open up the course book and because I had done my masters here, I had taken a lot of what they consider “easy” math down the hill so all that was left was combinatorics and graph theory and there was some other really really hard calculus. And I was like, naahhh I don’t really want to take the hard calculus. And I think that was it. I had to go to [a mathematics education faculty] for tutoring even [he] didn’t understand some of the stuff. But he helped me through with a B plus (Reflection, Mia).

Her sense of the purpose of the content of these courses was to acquire more of what Mia calls, “mathematics with a capital M”. (M)athematics, according to Mia, was the kind of math other people felt was important rather than mathematics that felt “meaningful or relevant to the mathematic I have to teach”. She attributed her (M)athematical experiences to, “the way it was taught” and the fact that she was in the, “student mode of survival”. Mia described this mode as feeling as if, “I need to do this homework assignment because it’s due. And I need to study for this test because its part of my grade. Not because I want to try and understand the mathematics.”

Faith’s recollection of the purpose of having to take mathematics course, as communicated to her by the mathematics education program, was to add more mathematics credits to her program of study. According to Faith, this was irregardless if the taking of the courses led to understanding of more mathematics.

It seemed as if the purpose of taking the 200 level calculus in my doctoral program was one of being tested. I had been given conditional acceptance into the doctoral program. Before being fully accepted, I would need to successfully complete the two series calculus course. Because of this, I think that the purpose in taking the class was always clouded by questions of my own mathematical worthiness. I wondered if taking the courses was truly about enhancing my own mathematical capabilities or was it to show a commitment by the center to ensure I was getting ‘enough’ mathematics. The purpose of the doing in the class was to learn or understand the mathematics that someone else had once done. This was obvious in the pages of the text and the way in which we were told about mathematics and then expected to practice it.

Wendy on purpose: Experiential descriptions.

Wendy's described her experiences in 200 level mathematics courses in her doctoral program as, "a means to an end". She felt frustration at having to take courses that she could not see as being beneficial in her goal of becoming a mathematics educator. She suggested that the reason offered from faculty about having to take such courses was to "beef up" her math background. However, she felt this was accomplished, "only on paper", as she didn't feel the courses were mathematically meaningful in her preparation of becoming a mathematics educator.

Wendy did not see purpose of taking mathematics courses other than recognizing taking the course as, "a means to an end" and admitted not wanting to, "put a lot of effort into it other than going home and doing my homework." The course did not engage or enthrall her, which she claimed was the case for most of her mathematical experience. She expressed disappointment that there was almost nothing in the experience that would help her on her journey of becoming a mathematics educator. Wendy felt as if her experience was about checking off a requirement, and added that she felt the purpose of taking the course was to prove her mathematical ability to be in the doctoral program.

Wendy claimed that she often wondered, "Why do I need to be here?" Her experiences were ones that allowed her to "just get through"- not at all the intended experience that any pedagogue would want for their own students. She shared the following about taking the Calculus series in her program:

I don't think I put a lot of effort into it other than going home and doing my homework. It wasn't like I was enthused about it at all. So, it was just a means to an end. But, I still don't know to this day, why I had to take it. What benefit? It didn't teach me anything about the math that I need to know even if I'm teaching the GED high school course; I still don't know what that will do for me (Wendy, reflection).

Analysis

Research has shown that teachers' beliefs about mathematics and teaching mathematics are formed mostly as a result of having been students in traditional mathematics classrooms (Thompson, 1992; Lortie, 1975). One of the responsibilities that mathematics educators typically have is to teach. Therefore it is critical to look at the ways that we, as mathematics education doctoral students, continue to be taught. Participants' experiences in *typical* mathematics courses are important to consider for this reason. The 400 level mathematics course is the most typical mathematics course taken by mathematics education doctoral students in this study. The majority of the 400 level experiences that these three participants shared illustrate classrooms that would be considered traditional.

Authority

Who was the accepted source of expert information or advice in the 400 level mathematics course that we were students in? What was the bible? What was the mathematical jurisdiction? Whose ideas were supreme? In this study, participants' experiences suggested the professor was the source of experience and advice. The course text was the bible. Along with the professor, it determined the mathematical jurisdiction. Students were seldom allowed to share ideas or ask their own mathematical questions other than the kinds to clarify that which was presented by the expert.

Participants spoke of the professor's role in their 400 level courses. Mia and Faith were in agreement that the instructor was the sole authority figure in the class most of the time. Mia's recollection of this authority was evident in the fact that the instructor "lectured for the entire class" and made all the decisions about what was to be learned

and how and when it was to be presented. Her role in the class was one of no authority. She was to ‘receive’ the knowledge that the instructor provided. Her lack of voice was a confirmation that the authority belonged to the instructor. Stephen Ball (1993) provides a vision of a classroom that is consistent with the Mia’s descriptions of authority in 400 level courses in which desks are “in rows, the children silent, the teacher at the front, chalk in hand, dispensing knowledge” (p.209).

The majority of Faith’s descriptions indicated that the professor determined what was to be learned, in what way, and by what pace. The professor’s authority was also in the assessment and grading systems in the courses. Unlike Mia who did not provide a counter example of authority, Faith was able to provide two examples of classes where the instructor shared some of the authority with students. In both of these cases, the instructor was more engaged with students and, “*invited* the class to solve problems/tasks as a means to construct knowledge and prove theorems...he didn’t just *present* the theorems to be learned.” Another aspect of the instructors’ willingness to share authority was in the professor’s actions when students in the class successfully proved a theorem. . Faith recalled the personally meaningful aspect of the class that when a class member who proved a theorem, they were then rewarded by having the theorem named after him/her. With respect to the authority in the Transitions course, Faith attributes the feeling of more collaboration than authoritarian to the fact that there was no textbook, and there was some flexibility in where the content seemed to go although we did follow a problem set.

The experiences that both Faith and Anne Marie shared are ones that offer students a voice, establish an intellectual community of learners rather than a strong-arm

knower-learner environment. These mathematical environments are need to increase mathematical learning and meaning- making (NCTM, 1991, 2000; Cobb, 1989; Wilcox et al, 1991).

Interaction

What was the nature of the interaction in *typical* courses? Was there evidence of collaboration, community, discourse, being and doing, sharing and being shared to? The “doing together” within a mathematics class can range from a scene of little interaction to a scene where teacher and student roles blur as both parties work together to make meaning. In this study, participants reported the level of interaction in their *typical* courses as being on a scale of very little interaction to being fairly interactive. The interaction that did occur in these classes was generally between instructor and students as well as between students but it was never reported that there was a deep interaction between students and the mathematics.

Mia reported that there was very little interaction in *typical* mathematics classes. In class, students primarily took notes while the professor lectured. The instructor-student interaction consisted primarily of questions being asked at the end of the class. The interaction in class was even less between students. According to Mia student interaction with the mathematics was that of a traditional class where students engage in activities such as drill, repetition, and memorization. The interaction involved the teacher telling the students about the mathematics and what to do with the mathematics and then students were expected to learn and do the mathematics. The line of discourse was generally one-way. Students rarely spoke to each other about the mathematics. This level of interaction is consistent with most traditional mathematics classrooms.

Faith's general characterization of the interaction between students in her 400 level classes was "practically non-existent" and the interaction between the professor and students as "students raising their hands to ask questions for clarification purposes (but this didn't happen often)".

In both of the 400 level mathematics course that Anne Marie took, the interaction was at a high level both between the instructor and student but also among students. In each course, the instructor would typically provide a brief introduction to a mathematical idea or facilitate a discussion about a problem, theorem, proposition, or mathematical idea. Instructors placed students in groups to work on a bank of examples. The teacher and student were not working together on the mathematics side-by-side.

Recent research indicates the importance of mathematics classrooms in which students discuss their mathematics and explain and justify their solutions (Yackel, 2001) and that students learn through opportunities to explain, justify, and listen to the ideas of classmates (Maverich, 1999). Both Faith and Anne Marie shared examples of 400 level mathematics classes (in one example, they were speaking of the same class) where students discussed mathematics together and were afforded opportunities to explain, justify and listen.

These more interactive experiences did not, however, resemble a picture of a classroom where students and teacher were actually working together on mathematics, which is an even higher level of interaction. This, according to (Wilcox et al, 1991) is likely to engender in students a very different view of mathematics from the typical rule-and-procedure orientation. The interaction in the class appeared to be for the purpose of encouraging students to communicate ("there was a lot more space for

discussion/dialogue with all members in the room”) and engage on some level of problem solving.

Mathematics

What kinds of mathematics did participants engage in within *typical* courses? What was the nature of the mathematical ideas? Whose mathematics was studied? What was considered important? Did our experiences provide the “broad and deep mathematical knowledge” as suggested by the NCTM and AMTE that mathematics educators need in their preparation? The descriptions of these ideas were consistent among Mia, Faith, and Anne Marie’s experiences in *typical* mathematics courses. Each mentioned the kind of mathematical content, how it was discussed, and whose mathematical ideas and questions were important in the class.

Participants described the mathematics content in their *typical* courses as having already been decided upon without much involvement of the students. Students received syllabi, texts, problem sets, and packets with the mathematics that they were to engage with. The instructor engaged students, in varying degrees, with mathematics that had already been done by someone else. Students were provided with the mathematical questions and problems and opportunities to solve and practice these. Students in these courses all worked on the same mathematics regardless of past experiences, ability, or interest.

Mia was clear in her descriptions that the mathematics she encountered was not the mathematics she felt she would be using in her future. Mia describes them as challenging, leaving her to feel confused, and having nothing to do with the mathematics she was to teach. Faith’s memories of these experiences were distinctly painful as a

person who had experienced mathematics in a very positive way up until graduate school. She described the mathematics as hard and recalled that; “I only really began to understand the mathematics in the course material when I worked individually with [the mathematics education faculty member]. Anne Marie recalled most of the mathematics in both the Transitions course and Abstract Algebra as being about, “solving and proving”. In both classes, students were given sets of problems that we were to work on both in and out of class. She remembered being often frustrated due to her limited background in proof. The mathematics that was studied was not about her own questions or conjectures. It was about someone else’s.

The mathematics encountered in *typical* mathematics classes that was mentioned by participants could be mostly closely classified as school or journal math according to Richards (1991). For Mia and Faith, most of their experiences were very much in alignment with school math culture. That is students were asked to solve mathematics in a habitual, unreflective way. The doing of mathematics this way did not actually produce mathematics or mathematical discussions. These approaches are reminiscent of Dewey’s description of “traditional” approaches that rely on “bodies of information and of skill that have been worked out in the past.”; therefore, fulfilling the main purpose of schools of transmitting that knowledge to the new generation (1938, p. 17).

For Faith, some of her experiences, along with Anne Marie’s, more resembled Richard’s description of Journal Math, which is considered more like the language of mathematical publications and papers. The emphasis of Journal Math is on formal communication and reconstruction which is more in alignment with the heavy emphasis that was place on proof in the course. In none of Mia, Faith, and Anne Marie’s discussion

of the nature of the mathematics were there descriptions about being students asked to actively "build" knowledge and skills (Bruner, 1990). There was no mention of students and teachers working on authentic, challenging problems together.

Purpose

What were the goals for students to take *typical* mathematics course? What was the nature of the motivation in doing the mathematics in the class? For most participants there was an agreement in their reflections about why they were taking the 400 level mathematics courses. It was in consensus that the purpose was "take more math, "check off the requirements" and "satisfy the mathematics ed faculty." Taking the courses was in part something they all had to do because it was important that they all get "more math" in their programs. Participants expressed concern and/or disappointment in the fact that the mathematics did not feel particularly relevant or meaningful. It bothered the participants that there was a disconnect in the learning of mathematics education with the experiences they had in the mathematics department. Participants were looking for opportunities to engage in mathematics in their 400 level courses that would help them to make connections to and more deeply understand the mathematics in the k-12 classroom.

The emphasis of earning mathematics credits as a credential for becoming a Steward of the Discipline is a theme that emerged across several participants. The demands of the mathematics education profession, the history of the field, the nature of the relationship between the field of mathematics and mathematics education may all be influences on why mathematics education doctoral students are advised and required to "take more math". For these participants, however, these *typical* courses did not meet

their personal and professional mathematical needs. The classes, from the participants' perspective did not broaden and deepen their mathematical knowledge.

Conclusion

This chapter reported on participants' descriptions of their mathematical experiences in courses classified as *typical* by the researcher. These initial descriptions were first presented through the use of the phenomenological tool of "bracketing" (van Manen, 1990). In participant's descriptions of their *typical* experiences there were similarities in the descriptions along the dimensions of authority, interaction, mathematics and purpose. The majority of *typical* course experiences that participants reported on described a course scenario where the professor was the authority figure in the classroom and engaged with students in a minimal way. The professor most often lectured and students were allowed to ask questions at the end of class. Students typically did not work together and the mathematics that was studied was not about students' questions or problems, rather the instructor and department predetermined it.

These descriptions were followed by experiential descriptions of these experiences and the meaning of these experiences for participants. "Experiential descriptions" uncovered participants' emotions, perspectives, preferences, expectations, and feelings about their experiences. Experiencing mathematics primarily and consistently in this "traditional" way during doctoral program was problematic and disappointing for participants, all mathematics education doctoral students.

In the next chapter, *novel* course experiences are presented in parallel format to the course experiences descriptions in this chapter on *typical* course descriptions.

CHAPTER SIX

Introduction

This chapter reports on participants' descriptions of their mathematical experiences in courses classified as *novel* by the researcher. That is, in participant's descriptions of their experiences, I found similarities in the descriptions along the dimensions of authority, interaction, mathematics and purpose in the experience that participants described as unique, or *novel*, in the mathematics coursework in their program of study.

The structure of the data presented in this chapter is parallel in structure to chapter five. Course experiences are shared through each of the four themes: authority, interaction, mathematics and purpose. First, a brief introduction to the theme is provided. These introductions pay particular respect to *novel* courses, and build upon the general theme descriptions provided in chapter four. Then, bracketed features of *novel* course experiences with respect to the theme are provided by several of the participants. Next, one selected participants' voice highlights the theme through experiential descriptions. Individual participants were selected by the researcher based on several factors including but not limited to the level and saturation of a participant's voice on a particular theme. The presentation of data is repeated for each of the themes. Lastly, an analysis and conclusion are provided.

All six participants in the study were required as a part of the doctoral program to take 400 level mathematics courses taught within the mathematics department. Four participants took either one or two semesters of the 788 *Foundations of Mathematics* (FM) course, described earlier in chapter four. The *novel* course experiences that are

discussed in this chapter are from select 400 level mathematics courses offered in the mathematics department and the 788 *Foundations of Mathematics* course offered through the education department.

Novel Mathematical Course Experiences

One group conversation in this study was primarily focused on having participants share their mathematical courses experiences with each other and to collectively think about these experiences with respect to their overall preparation of becoming mathematics educators. During this conversation, I asked participants to imagine a university that did the best job at preparing mathematics education doctoral students. At Utopia University, graduate mathematics classes for mathematics education doctoral students would be taught in accord with current research on best practice in mathematics education. The content and pedagogy of the course would embody the spirit of the discipline of mathematics that we had come to envision through our professional study of mathematics education. I asked participants to describe the reactions that mathematics education doctoral students might have to such ideal mathematics courses designed and taught at Utopia University. The following are a representative sample of their written descriptions to my prompt:

- *We were really “doing” math.*
- *Even if I don’t know the math right away or how to answer a problem, I have the confidence that I could work through the problem.*
- *I now feel that mathematics is really creative and that asking questions is a part of doing math.*
- *Math makes sense!*
- *I actively participated in developing my understanding of math - it wasn’t just someone else’s knowledge.*
- *My math experiences were so meaningful and engaging.*
- *I revisited the math that I plan to teach and I revisited it on a much deeper level.*
- *It was empowering in the sense that I felt like I was important in the development of ideas.*

- *It was student centered and a model for how I want to teach.*
- *We worked together- both teacher and students.*
- *I was totally exhausted by it because I always wanted to think about the math.*
- *It was useful and it helped me to revisit and think more deeply about “simple” mathematical ideas.*
- *We made connections.*
- *We did a lot of projects.*

Although no participants claimed that their actual mathematical experiences would be described as purely Utopian, there were course experiences that participants had that resembled the kinds of Utopian descriptions provided in our discussion. These *novel* experience descriptions provided a stark contrast to those experiences described as *typical*.

Authority

In *novel* course experience descriptions, the nature of authority was the idea of who offered advice, opinions and influence in class was mentioned consistently. That is, when speaking of their perceived experiences, participants suggested that students in these *novel* course were encouraged to invent ideas, ask questions, offer opinions and advice, and generally influenced what was important in the class. All the participants spoke to the nature of authority in *novel* mathematics course situations as being shared between students and instructor and provided images of reform inspired mathematics classrooms.

Bracketed experiences about authority.

Faith provided descriptions of two courses in which the instructor was engaged with students and, “*invited* the class to solve problems/tasks as a means to construct knowledge and prove theorems...[the instructors] didn’t just *present* the theorems to be learned.” These experiences were in contrast to the majority of the courses she had taken.

While many of the traditional elements one might expect from a mathematics course were still present (homework, tests, quizzes, teacher and text book as the main sources of knowledge), the instructor created engaging problems/tasks that he invited the class to solve as a means to construct knowledge and prove theorems. In other words, he just didn't present the theorems. Faith contrasted this to *typical* course experiences where the instructor would merely present theorems and students would recreate them. According to Faith, the instructor created tasks that evoked thinking such that somebody in the class could prove the theorem. Faith detailed a personally meaningful aspect of the class; when a class member who proved a theorem, they were then rewarded by having the theorem named after him/her.

The Transitions to Advanced Mathematics course (described in chapter 4), was a second course where Faith reported as experiencing authority in a different way. The Transitions course was taught by a mathematics education faculty, who held a joint position in the mathematics department and education department. The course was unique on many levels: the size (there were only three students), the nature of the content (the course was developed specifically with elementary mathematics education doctoral students in mind versus being a prepackaged course and an element of a regular line up of mathematics content courses), the fact that it was taught in the education building and not situated within the mathematics department, there was no textbook, and the flexibility in where the content seemed to go. Faith shared that it seemed that the instructor had a general direction and big objectives he wanted to students to study but that he seemed to, "go along with the flow and was figuring things out as we went." Faith perceived the authority to be shared among the teacher and students as, "there was a lot more space for

discussion/dialogue with all members in the room” in addition to the fact that the students, “had a lot of opportunities to ‘go to the board’ and it didn’t seem as frightening as what might have been the case if I had to do this in any other mathematics course I’ve taken.”

This sort of flexibility was evident in the FM course. When Kristen compared her experiences in the FM courses to those of her other 400 level mathematics classes, she shared that in the 400 level courses typically the, “professor and the textbook were the sole sources of authority” and the students’ voices were minimal in class sessions. In her descriptions of FM, Kristen acknowledged that the students had a “good deal of authority” but that, “We looked to them (literally) when we had mathematical questions.” This was evident in the mathematics that was explored in the course as, “Individuals would do explorations on their own [outside of class] and present them to the class”. As these presentations occurred, students were charged with posing questions to each other and engaging in mathematical conversations.

Kristen also alluded to the shared authority that was felt in the FM class in her descriptions about the not-knowing of things in class. She shared, “I felt comfortable admitting things that I didn’t understand.” This, according to Kristen, was unusual for a college mathematics course. Kristen also discussed that fact that a lot of the mathematics that was discussed and explored in FM had been mathematics that she had seen, “at some point in my mathematical career.” But, in those places, she had merely “touched upon” them and most of the ideas were, “things that I was told.” For example, she remembered being told that there were different sizes of infinity and had memories of just having “to accept it and move on” even if, “it never really truly made sense.” Kristen felt that the

FM classes afforded her the authority to say, “You know what? That didn’t really make sense to me back then and I do have this misconception between my intuition and what I know to be true because it’s in the math books.” Kristen admitted to not feeling entirely confident in her math ability, but she was certain that in the FM course she, “did feel confident that I asked the questions and that we would struggle with them.” In describing her not-knowing, she recalled, “I can’t say that *everything* we did made sense to me, but that I definitely thought that I should keep asking questions because I think it should make sense and I don’t have to just accept it.”

Eva also reported on the authority in the Foundations of Mathematics course as begin *novel*. According to Eva, “student control was central” in the FM course and was exemplified by the important role of argumentation and justification among students. This aspect was unlike the majority of Eva’s other mathematical experiences where Eva acknowledged the teacher and books as the sole sources of authority. Eva shared that “student control was supported and developed” in the course and that “individual, group and instructor enquiry” were all present. One of the examples that demonstrated shared authority, according to Eva, was that each student was required to select a problem that either developed from the course content or from previous experiences explore the problem situation outside of class as a major portion of the course requirements.

In Eva’s recollections of these projects, the students dictated the forms of the explorations in most cases, and as a result, the students were very excited about what they learned and the understandings that they developed. According to Eva, “To a certain extent the course was about personal sense making and exploring as a group.” In the following reflection, Eva describes her perception of the nature of authority in FM.

The authority in this class was shared by all, at least all of those who chose to accept this authority. While the professors had the authority for bringing some of the problem to the class, and choosing to present material that we might find helpful, we were able to pursue lines or reasoning, experiments, solutions, etc. that we felt would be helpful. There has been some debate as to how much [the instructors] directed the class, and I think in many cases they did help, but I also think it was useful direction... for example I do not have the idea that had we pursued an alternative end than they had thought of, that they would force us to follow them. I think they had ideas about how to help us make something productive and helped us along from time to time, but in most cases any direction was useful to us at the time, and wanted and therefore supportive of us, rather than their exercising authority (Eva, reflection).

Eva described feeling the shared authority, “maybe even a bit more in the second semester of FM as we each had crystallizations that we brought to class from which questions were raised.” The crystallizations were a way to formalize and summarize some the mathematics that had been done in the previous class session. Individual students in the course took the responsibility to write up a crystallization after each class and shared them with their peers in the next class session as a way to take ownership of synthesizing the groups’ mathematical progress and frustrations.

The notion of shared authority rose in Eva’s description about a specific mathematical example that occurred in class. The students and instructors had been working on the notion of “squareness of rectangles” and students were “a bit stuck in the same old ideas,” and [the mathematician] pushed us to broaden our ideas by suggesting her own mathematical ideas and knowledge to get us to think about squareness in “her” way to shift the discussion and the mathematics in a particular way. However, she [the mathematician] never forced us to prescribe to a particular measure.

The instructors’ authority, according to Eva, “Came at the right time for us, was never too early, was rarely unsolicited, and was never because they were not happy with what we were doing. In that sense the authority was really shared.” Eva claimed that the

instructors "directed" when "we wanted them to. That, I think, is what made the instruction so good."

My own experiences in *novel* courses were similar to those mentioned by Faith, Kristen, and Eva. I experienced three courses where the authority felt shared between the students and the instructor. In the Transitions to Advanced Mathematics course, the instructor provided the three students with problem sets, theorems and propositions that were to be worked on over the course of the semester. The instructor facilitated most sessions by setting up problems for students to work on but it was expected that each student share their work and ideas during class time. Each student went up to the board to share proofs to the propositions.

In Abstract Algebra, the professor's role was similar in that he did not lecture but rather presented mathematical ideas, and showed examples of student work. During group work, the instructor and teaching assistants monitored students' progress and asked probing questions of groups to help clarify their ideas or to help groups that were "stuck". At the end of most class meetings, group representatives would share their ideas, or proofs of propositions that they examined during the session. Other students were charged with examining the work of the group that was presenting and raising issues, questions, or comments that would help students to synthesize their understanding of the mathematical ideas. The professor and T.A.s also worked together to decide what material would be on the tests. The grading system was unique however in that it worked on a sliding scale. Students had some authority to determine (up to a point), how many percentage points each particular portion of their grade would be worth.

A third course experience that I recall where the authority felt shared was in the FM courses. The authority was shared in deciding what mathematics would be studied and whose questions were important. I describe the perceived sense of shared authority in the following journal reflection:

Our mathematical experiences and questions became the topics of discussion on the syllabus. (They were the unscheduled schedule of topics.) Starting with our own questions sent the message that the things that were truly important in the class belonged to us- our questions, our problems, our mistakes, and our discoveries. All ours. No textbook. No teacher's manual. I do recognize, however, that although it may have felt that the instructor's role was minimal, for this to happen, it took a great deal of planning and preparation and skill (Anne Marie, reflection).

Anne Marie on authority: Experiential descriptions.

In the Abstract Algebra course I took, it was my experience that the instructor did not lecture but rather presented mathematical ideas, conjectures, and theorems and showed examples of student work. There appeared to be a certain level of authority in the instructors and teaching assistants (T.A.s) as they were explicitly in charge of what content was to be learned and discussed and also had the "answers". Although there was student autonomy in the structure of the class, I very often found myself looking to the authority figures of the course for help in solving the problems and getting *the* right answers.

In the Foundations of Mathematics course, I sensed an even greater sense of shared authority. This took some time to get used to. Having a "voice" in the class was unlike most of my previous mathematics course experiences. Having a voice was about being able to ask questions and challenge authority about the mathematics and pedagogy that we were experiencing as well as how things were going in class.

I had been in other classes where students asked questions. But the questions typically were about the mathematics that had been done by others. These questions were reactions to the mathematics presented to us or the mathematics we had worked on in class or for homework. The questions in the FM courses were questions that as inventors of mathematics, not customers.

In our end-of-class journals I recall asking, “Why Bother?” as a deep desire to know why the class was engaging in certain mathematics and what it would do for us as learners of mathematics. I believe that the authority and openness of the classroom allowed me to say, “I don’t get it” or “I don’t like it”. I felt my questions and comments weren’t an avenue to complain, but rather a meaningful way to say, “I know my opinions and experiences are valued and here is what I have to say.” In an end-of class journal, I wrote:

I haven’t been a fan of the end-of-class journaling for one reason. Sometimes I worry that I won’t have anything worthwhile to say. Sometimes there is so much going on in my head I don’t know where to start and 15 minutes is barely enough time to figure it out. But having to write a reflection for today has been the biggest challenge by far. I didn’t like today. And I don’t know if I didn’t like today because it felt unproductive mathematically or I felt especially challenged or I was annoyed with people. I felt lost a lot. I felt angry sometimes. I was sensitive to the fact that how I know certain things may very well be quite elementary and I was afraid to step out of that box a bit. Nevertheless today felt strange.

I think it seems appropriate and productive that we began class again by listing some of our own questions on the board. We always have exuberant amounts of questions generated in class but the difference between today and other days is that I don’t really feel we got remotely close to really answering any of them. Not answer in the traditional sense but rather making sense out of things. At least for me. I left feeling dissatisfied in my understanding of point, line, measure, length, dimension, counting....

For example, take POINT. I was toggling back and forth in my head with this little voice that I wanted to shoot. Because I wanted to think about point in the way that I know. A little dot. There has never been a reason in my mathematical

experience to ever think otherwise. It's always worked for me. I can see it. It takes up space. Even the smallest imaginable dot has dimension. It's something. (Versus nothing). But the more I was annoyed today; my understanding of point seems to fit less and less. But how can that be? And why am I just finding out now? So, part of my anger and dissatisfaction lies in the fact that I like my idea of point because I'm comfortable and it works in my world but all of a sudden there are some things that aren't working out for me and I might need to start thinking about it in a different way. And letting go of the certainty of something that felt so right, real, mathematically correct, obvious, etc., forever now is difficult.

One thing that I'm not ready to accept for sure is the notion that if I added up all the irrationals (measure being limit 0) that they would equal 1. That is ridiculous. I'm quite sure that it's connected to the way I'm not thinking about points but there seems to be more. And I hate that someone would maybe agree that this seems reasonable without batting an eye. Your gut has got to be hurting about this one. Anyway IF this were true, it seems to be a great big supporting block for my .9 repeating argument. If those limit 0 pts are being added up to 1 then there must be a little bit of something (I don't care how small) that is getting added together. Limit 0 is not the same as 0.

And then, when the formal definition came out of that book. I was really turned off. Besides looking and sounding like a foreign language, I couldn't see how it was supposed to help. Did the definition clear things up for people the same way the Cauchy sequence cleared things up for us last semester? I don't think so. Would this be what a mathematician would call a bad day at the office?

The emotion revealed in this reflection highlights the frustration and excitement that occurred due to the shared authority in this way of experiencing mathematics. A unique aspect of the authority in the class was not having to solve a predetermined set of problem coupled with a sense of freedom to ask questions and express ideas and feelings about mathematics that was personally relevant. My growth in coming to understand and accept a mathematical idea was nurtured in this safe environment where I was expected to play with mathematics and experience mathematical growing pains.

One of the examples of shared authority in the FM courses was the omission of textbooks, quizzes, and tests. I recall thinking, "Everyone plans their semester around the

weeks that quizzes and tests are given. How would they (the instructors) know what I knew? What I didn't know? How would I know what I knew? What I didn't know?" Not having these artifacts in a mathematics class affected the authority felt by students. The omission of tests and quizzes sent a message to me- that I wouldn't need to be spending my mathematics time studying and cramming for an exam. Instead, I would be spending my mathematics time thinking and doing and that these mathematical activities were valued.

My experiences in FM were transformational. The courses helped me connect and learn mathematics in a deep and meaningful way. This was important to me, especially considering my "elementary" background which had been a point of contention in my acceptance to the doctoral program. I felt fortunate to have taken the courses, as they were not like any other courses I had taken. The opportunity to connect this way of doing mathematics with that which I had learned about in my other coursework and shared with my students in courses I taught, such as elementary mathematics methods, was invaluable. For me, what seemed particularly important was the way in which the wide range of mathematical understanding, experience, and ability, was embraced and used to the advantage of the group as a way to explore mathematical ideas from many angles.

Interaction

The notion of interaction was another theme that emerged from the data. The interaction theme in *novel* courses, as described by participants, often provided a picture of classroom communities that strived on "doing together". This included collaboration, mathematical discussions, being and doing, and sharing and being shared to. Teacher and student roles often blurred as both parties worked together to make meaning. The "doing

together” referred to the interaction between different classroom relationships: between teacher and student, among students, or between the students and the mathematics.

In *novel* courses, the interaction between teacher and student was consistently described as two-way. Participants described students in these *novel* courses frequently initiating a mathematical interaction with other students or the instructor. Participants reported little to no lecture in *novel* courses as well as high levels of participation among students in class sessions. With regard to the interaction with the mathematics, participants provided several examples of students interacting with the mathematics in meaningful ways.

Bracketed experiences about interaction.

Faith referred to her number theory course experience as providing, “more interaction with students in this class compared to other mathematics classes.” Faith shared that, “while in most cases the interaction occurred between professor and students, if a discussion got pretty interesting, more class members participated by chiming in both to the professor and to other students as well.

Faith referred to the Transitions course and her Number Theory course as examples of contrasting experiences to *typical* experiences. With regards to the Transitions course, she recognized that the level of interaction was partly attributed to the fact that there were three students in the course and, “we were peers rather than strangers.” In this class, the instructor kept the environment very informal. “It was like a conversation”.

Kristen also spoke to the interaction in the *novel* courses she experienced. The majority of Kristen’s comments about the interaction in the FM courses were focused on

the idea that the students in the class had such different mathematical backgrounds and strengths and the courses appeared to reach all the students in meaningful ways.

Kristen recalled that the mathematical interaction in FM was often ignited when, “[The instructors] would begin an exploration by posing a problem. Then we, the students, would brainstorm solutions, related questions, and ideas. We’d share these ideas with each other.” She shared that the instructors interacted with each other and the students constantly. In addition, most of the “work” that occurred in class was among students and not merely involving students “copying notes” and taking in what the instructor said.

Kristen provided a very specific example that illustrated how the themes were different in FM than in other classes. She talked about a particular topic that we had worked on in FM for several weeks. It started with a prompt about determining the “squareness” of rectangles. She shared how we had worked together, students and instructors, on ways to order rectangles. For multiple weeks, we explored and shared strategies, compared strategies and eventually “came to” continued fraction sequences as a way to compare the squareness of rectangles. This, Kristen recalled, was drastically different from the one other experience with continued fractions she had where in one class the, “professor made mathematical statements and gave examples showing these ideas at work...and in about 20 minutes on the last day of class he gave a very quick overview of continued fractions. At the time I remember thinking how different his explanation was to our exploration of continued fractions in FM.”

I also described the interaction in the Transitions, Abstract Algebra, and FM courses as *novel*. It is my perception that the level of interaction was at a high level both

between the instructor and student but also among students in these courses. In most class sessions, the instructor would typically provide a brief introduction to a mathematical idea or facilitate a discussion about a problem, theorem, proposition, or mathematical idea.

In the Transitions and Abstract Algebra course, the instructor placed students in groups to work on a bank of problems. The instructors provided propositions and students would evaluate and prove the truthfulness of the propositions.

In the FM course, the mathematical exploration and interaction was often led by student questions. Progress was mainly generated by students working together. During FM class sessions, there seemed to be long stretches of time when neither instructor's voices were heard. It wasn't as if their voices weren't crucial to the class, but what was clear was that students' voices were just as important.

Faith on interaction: Experiential descriptions.

There were two specific course experiences that provided memories that contradicted Faith's *typical* description of interaction within mathematics courses. First, Faith referred to a number theory course experience as providing, "more interaction with students in this class compared to other mathematics classes." Faith shared that, "while in most cases the interaction occurred between professor and students, if a discussion got pretty interesting, more class members participated by chiming in both to the professor and to other students as well. This experience was so powerful that Faith committed, "Experiencing learning in this way motivated me to think about how I may incorporate some of these experiences into teaching school mathematics to young children and to teaching pre-service and in-service teachers of mathematics." Specifically, she

acknowledged the course being a useful example of crediting young students' thinking by naming strategies or proofs after them and then having the class refer to these as resources to draw upon in future problem solving tasks. "I think for a second grader to experience something like that it conveys the idea that they can view themselves, their peers, and not just the teacher as the source of understanding and meaning making."

While many of the traditional elements one might expect from a mathematics course were still present (homework, tests, quizzes, teacher is main source of knowledge along with text book), the instructor created engaging problems/tasks that he invited the class to solve as a means to construct knowledge and prove theorems. In other words, he just didn't present the theorems. He created tasks that evoked thinking such that somebody in the class could prove the theorem. Faith detailed a personally meaningful aspect of the class that when a class member who proved a theorem, they were then rewarded by having the theorem named after him/her.

The second contradicting experience of authority within 400 level mathematics courses was of the Transitions course. The course was unique on many levels: the size (there were only three students), the nature of the content (the course was developed specifically with elementary mathematics education doctoral students in mind vs. being preconceived and part of some regular line up of mathematics content courses), the fact that it was taught in the education building and not situated within the mathematics department, there was no textbook, the flexibility in where the content seemed to go. Faith shared that it has seemed that the instructor had a general direction and big objectives he wanted to students to study but that he seemed to, "go along with the flow and was figuring things out as we went." The authority was definitely shared among the

teacher and students as, “there was a lot more space for discussion/dialogue with all members in the room” in addition to the fact that the students, “had a lot of opportunities to ‘go to the board’ and it didn’t seem as frightening as what might have been the case if I had to do this in any other mathematics course I’ve taken.”

Mathematics

The mathematical content, how it was discussed, and whose mathematical ideas and questions were important, define the *mathematics* theme. In *novel* courses, the mathematics was described as being “creative”, “being about students’ questions”, and “relevant”. The descriptions resembled original origins of the word mathematics as being related to *manthanein* “to learn,” from the base *mn*, *men*, and *mon* meaning “to think, have one’s mind aroused”.

Bracketed experiences about mathematics.

One of the unique mathematical experiences that Kristen recalled in FM was that the doing of mathematics felt “creative” and that it was the kind of mathematics that inspired her to think about mathematics outside of the class sessions. At the time of the interview, it had been two years since our first semester of FM, and Kristen admitted to “continuing to think about things like the problem I worked on the first year- the multiplication problem”. Kristen’s thinking about the problem and her experiences motivated her to continue to work on the project that began in semester one. She continued working and editing and submitted it to a mathematics education journal. “And I definitely thought about it outside of class.” According to Kristen, another unique and important part of the FM course experience was the research component, which required students to develop a question and research/explore it at length throughout the semester

with the notion that students would formally present their project progress to each other throughout of the semester.

Eva referred to the majority of her *typical* mathematical course experiences as being “individual” experiences. She claimed that most of the mathematics experiences were unmemorable and the mathematics was something students were expected to learn on their own. Eva’s descriptions of her mathematical experiences in the FM courses were much different. She claims having “done math” in many other classes but the mathematics in the FM courses, “was not at all like any other math experience.” One particular way that Eva described the mathematics as being different was, “the way we came to know mathematics, the activities, and conversations. What we considered to be mathematics was unique.” According to Eva, “We did not engage with problems in the ‘traditional’ sense”. Specifically, the mathematics we engaged with was not presented as a list of topics to be memorized or theorems to be proved. The mathematics we engaged with was about questions, ideas, and work that emerged from students in the class.

According to Eva, “the math, in most cases was about ideas and connections, not about something very specific to learn and then apply later. The math ideas were always accessible to everyone, but also were so deep that we could take them very far.” She recalled that students in the class, “all had a great deal of experience to draw on to make sense of these problems.” She described the starting problems, “with something more or less concrete that we could play with or visualize and this playing is what we anchored some of our arguments in. “Through play,” Eva posited, “the math became personally relevant.”

Eva described the way students engaged with the mathematics in the following description:

During class, the instructors often picked an area of exploration for the class as a whole. However, from here, enquiry was very open-ended. It was left to each student or group of students in the class to then investigate situations in a manner that made sense to them. For example, when looking at the area-perimeter investigation, some students explored how holding one dimension of the rectangle constant impacted the relationship; other students considered how changing units would impact the area-perimeter relationship; one of the instructors investigated the boundary of the relationship when area is larger than perimeter and vice versa.

Here Eva describes one of the big mathematics investigations being completely uncovered from students' questions:

The instructors selected the topic of real numbers. However, rather than presenting a problem situation, the students brought in various questions they held about real numbers. Once these questions were raised, students were able to work on particular ones of their own selection and investigate relationships. Some issues that were raised and investigated were the sense that there are more irrationals than rationals, even though both are dense; that certain irrationals have a particular characteristic of a repeating growth pattern (such as .1001000100001...) and what was special about these numbers. Two challenging issues that came up (one of which was addressed but not resolved, the other of which had many students not knowing where to begin) were whether .9 repeating equals one, and how we know that $\sqrt{-1}$ is not on the real line. We read a couple readings, like Zeno and Hotel Infinity, to help us begin to explore some of the situations and then used these to try to work on our own questions. The math then was relevant; the stories were of interest to us as a way to explore, while also offering ways to access the math arguments without symbols. As we closed this investigation, we actually returned to our own questions and were able to in small groups choose a couple of our own questions to continue to work on and try to answer, from the experience we had gained... many of which we realized we actually felt we had answered in our playing and investigations (not formally, but at the level of our own understanding of the math) (Eva, reflection).

Like Kristen and Eva, it was also my perception that the mathematics that arose in the FM courses was unconventional for a college level mathematics class partly because the content was often driven by student ideas and questions. The instructors would

propose a problem or conjecture or problem to the class and then students would have opportunity to think and work together as pairs or in small groups. From these small working groups, many different ideas and connection and questions arose depending on the students who were in the group. Students came to know who would think about problems algebraically or geometrically or make connections to their classrooms (k-12) and that each of these ways of seeing and doing seemed to lead to more and more opportunities to ask and do.

From my perspective, the FM courses were also unconventional in that the mathematics we investigated grew from the participants in the class. It was very often students' questions and inquiries that fed and led the mathematical explorations. In addition, there was less focus on mathematical procedures and techniques and more emphasis on building understanding and making connections across mathematical topics.

Kristen on mathematics: Experiential descriptions.

Many of Kristen's comments about the FM courses were focused on the idea that the students in the class had such different mathematical backgrounds and strengths and yet, the course was able to reach all of the students in meaningful ways.

The thing that really stands out for me in that class [FM]...in our math education classes, we always talk about how people, students with different backgrounds can all work together and learn together and I never really *believed* that or *understood* how that could happen. And our Foundations of Math classes, it was the first time that I ever really believed that everyone could be challenged and learn and we all had such different backgrounds. And I just thought it was amazing that I was challenged and everyone was challenged and I really enjoyed it. So, that's the number one thing that sticks out for me (Kirsten, journal reflection).

Kristen shared that the mathematics she experienced in FM was unlike the mathematics she experienced in other courses in that the mathematics in most other

courses, “existed before us – there was nothing for us to discover. We were to memorize and apply theorems.” Kristen expounded on this idea in her description of the mathematics she experienced in FM:

Although we didn’t discover any new mathematical ideas, it felt like we were discovering new ideas. The first semester I remember being really excited about the connection between the perimeter/area problem and the arithmetic/harmonic means. It really felt like we had discovered something new. I was so excited. I remember I brought it to [the mathematician] at some point out of class and said, “Is this new? Did we totally discover something?” And she was like, “Well, no...this has been thought of before.” I was heartbroken, but she told me too that she was so sad that she had to tell me that it had been discovered before. But it was new to us and so is that doing math because it was new to us or does it have to be new? (Kristen, reflection).

According to Kristen, another important part of the courses experiences was the research component, which required students to develop a question and research/explore it at length throughout the semester with the notion that students would formally present their project progress to each other throughout of the semester. In a reflection written for the course about the research component and the pursuit of asking your own questions and posing your own problems Kristen wrote:

The research component of this course allowed the students the time and support (space) necessary to focus on their own questions in a way that none of us had ever experienced before. As a student, this was both liberating and frustrating. It was wonderful to be able to focus on questions that were of interest to us and that were relevant to our own teaching. It was also very challenging because in our past experiences as both students and teachers of mathematics, the question were always given to us. Developing and working on one’s own questions is a very different, and often maddening experience. Through the process of posing and solving our own questions, we experienced some of the ups and downs that mathematicians experience.

In my experiences in other math classes, I had a sense of how to solve a problem simply by looking at when in the course the question was posed; the proof required only the theorems on the previous pages. Because the teacher assigned the questions, I trusted that I had all the knowledge and skills required to answer the question. In this course, since we were asking our own questions, we did not

know if we had the prerequisite skills necessary to solve the problem, or if the problems were even solvable. In math courses, students often look in the back of the book or other textbooks to find the solution to a question assigned by the instructor. Since we had posed our own questions, we could not do this and the students in the class purposefully avoided known solutions.

Although it may have been naïve, at times of exasperation, some of us [in the FM course] wondered if we had stumbled upon another great question of mathematics (like trisecting an angle) that will require years of work by the greatest mathematicians to solve or prove unsolvable. In fact, one of the students said that while he was working on his problem, to him, his problem *was* one of the great problems in mathematics. Our problems may not have been *real* problems in the sense of being unsolved in the mathematical field, but they were very real to us because we had posed them and did not know if anyone else had answered them. These experiences provided an avenue to experience the joy of posing and solving questions that matter to us (Kristen, reflection).

Kristen recalled positive learning experiences and shared, “I don’t think very many people have experienced anything like it. So, I feel really lucky.” In thinking about the course, Kristen acknowledged that the uniqueness of the experiences, as well as the students’ and instructors’ roles in making it unique:

They [the instructors] knew us so well and the group of us- we were so interested in mathematics and how you learn mathematics. I don’t know if it’s possible to recreate it with regular people [people other than mathematics education doctoral students]. But all of us care about the why and so many people don’t. like, my husband. He’s an engineer. He’s good at mathematics, he took a lot of mathematics but he doesn’t care at all why anything happens and whenever I’m all excited about ohh, I found another proof of division of fractions, he’s like, who cares? And so I think certain personalities get excited about that sort of thing and want to know why and want to know even more than one way of looking at it and most people don’t care at all or just want to find one way of looking at it. So they’re not going to be asking the kinds of questions we asked. I remember, when you asked, why bother? I was like, its fun (Kristen, reflection)!

For Kristen, the FM courses proved to be her “most memorable” course experiences in her doctoral program. The classes were a way to “see” how a mathematics course could be taught successfully with students that brought varying mathematical backgrounds. She acknowledged not having thought much about her mathematical

experiences in her coursework until she took FM which allowed her to see “how things could be so different”. This indicated her assumption that mathematics courses, particularly those in higher education, follow traditional paths with respect to the students’ role, teachers’ role and the role of mathematics.

Kristen recalled feeling excited in FM by the fact that the mathematics was creative and, “it felt like we were discovering new ideas.” These new ideas, even if they weren’t undiscovered bits of mathematics, felt to Kristen as if her curiosity and mathematical prowess had led her to new waters.

Purpose

The theme of purpose uncovers the intended or desired result, goals and makes clear the point of particular mathematical experiences. The participants’ notions of purpose were related to the purpose of taking the class, purpose of the content, and/or the purpose of engaging with the mathematics. Richards (1991) describes purpose in mathematic classes as being either “reason- giving” or “reason- seeking”. When reporting on *novel* course experiences, participants often reported an important purpose for being a learner in the class. They also described the interaction with the mathematics as having purpose. When participants mentioned the purpose of the mathematics in *novel* courses, the distinction most closely resemble the “reason-seeking” description made by Richards (1991). This theme is particularly distinct in that participants often communicated a powerful sense of purpose in *novel* courses, which is in stark contrast to their reported sense of purpose in *typical* courses which was weak and superficial (i.e. checking of a requirement), if mentioned at all.

Bracketed experiences about purpose.

In Eva's initial descriptions of her experiences, she never came forth to say why she felt she was taking the Foundations of Mathematics courses. When I asked her about this later, she suggested that knowing both the instructors she was excited about the possibility of what the FM course might be. Then, once she had taken the first FM course, she was more than eager to take the second.

In her recollections, she referred to a purpose that was enacted in doing the mathematics as not just "doing math" but also thinking about why what we did in the class was considered "doing math". She describes the FM courses as the kind of math class where, "a kid can just investigate problems and ask questions". This, according to Eva, was a class where she was allowed and expected to "think mathematically". According to Eva, being a learner in the class helped her to, "gain a sense of how you would actually go about investigating math."

Throughout her descriptions, Kristen alluded to the purpose of the mathematics and experiences in FM as being related to having, "an avenue to experience the joy of posing and solving questions that matter to us", "seeing it [the kinds of mathematics that mathematics education students learn about in their education classes] in action", and opportunities to "explore problems that we were developing on our own." The class provided Kristen an example of how things in a mathematics class, "could be so different" [from typical mathematics courses].

According to my program of study, I was required to take the FM course to fulfill a requirement of the mathematics education program. In the fall of 2003, I enrolled in an early iteration of the Foundations of Mathematics course. I dropped. After the first exam,

it was clear that the course was not compatible with my mathematical needs at the time. In the fall of 2005, a new Foundations of Mathematics was offered. Knowing the instructors well, I was confident that I would be supported to take the course.

In thinking about the purpose for the things we did in class, I feel strongly that our purpose was to be doing mathematics together in ways that a mathematician might. For me, the FM courses were less an experience of testimony that this kind of mathematics teaching and learning could occur, but rather an opportunity to do mathematics in a way that I had learned about but had not had opportunities to do. It was a course that allowed me the freedom to feel valued mathematically. It was a place where I felt that her questions and interests mattered, and my not-knowing wasn't a character flaw to be fixed rather a strength to be acted upon.

Eva on purpose: Experiential descriptions.

When speaking to the interaction in FM, Eva shared, "In general, I think people were very open to other peoples' ideas and questions and wonderings, and were willing to talk about them in a respectful manner. You could tell how people listened to each other because of how people's ideas built off of each others, and how one thing that one person said would allow a connection to be made (and also sometimes how what one person said really confused someone else as they tried to make some sense of it)."

In her recollections, she referred to a purpose that was enacted in doing the mathematics as not just "doing math" but also thinking about why what we did in the class was considered "doing math". She describes the FM courses as the kind of math class where, "a kid can just investigate problems and ask questions". This, according to Eva, was a class where she was allowed and expected to "think mathematically".

According to Eva, being a learner in the class helped her to, “gain a sense of how you would actually go about investigating math.”

Eva also shared that her experiences in the FM courses were pivotal in her thinking about how to provide these experiences to others. The experiences have since helped in her own research projects and thinking about the way she plans and teaches students in her preservice and inservice courses. According to Eva, “Without those classes, this [dissertation research] just wouldn’t be and I don’t think my teaching would be what it is. Eva shared that it was her desire have opportunities to teach courses in the future that would provide the students with similar mathematical experiences to those she was afforded in the FM courses.

Eva was adamant that her experiences in her FM courses were different from other mathematics courses she had taken in the past. She mentioned how different aspects of authority, interaction, mathematics, and sense of purpose felt in FM. The authority was shared. The interaction was consistently active between student and instructor, among students, and between students and the mathematics. The mathematics that was studied was about student questions and ideas and the purpose of the mathematics was to be *doing* mathematics.

In general, Eva’s experiences in FM were critical in helping her to have the opportunity to learn how to ask her own questions and seek her own answers to her questions. Although she had had some positive experiences in other mathematics courses, the Foundations of Mathematics courses were the most pivotal mathematics classes she had taken and afforded her opportunities to continue to think about how to do this kind of teaching.

Eva commented on the sharing and being shared to which was evident in how people listened to each other and then how people's ideas built off of each other's. She shared, "One thing that one person said would allow a connection to be made." However, this high level of interaction wasn't always a positive experience for Eva. She recalled two specific interactions that left her feeling really uncomfortable. In both of these instances, she attempted to share some ideas or conjectures and felt that the feedback from her classmates rejected them and thus violating her sense of "safety" in the classroom. These instances created a buffer for Eva as she claimed feeling "less free" to share the next time. However, it was clear that these instances did not leave a lasting impact on Eva as she claimed that she soon let go of her uncomfortableness as soon as she "got into more math." This may be testimony to the level of the sense of community in the classroom or it may also indicate the importance in the nature of the interaction with the mathematics, which called her to continue to engage in the way she did before the spells of being uncomfortable.

Analysis

Participant' experiences in the courses identified as *novel* are important to examine as they provide examples of mathematics education doctoral coursework that may strengthen knowledge, beliefs, and dispositions about mathematics content as well as about mathematics teaching and learning. Lee Shulman, former president of the Carnegie Foundations for the Advancement of Teaching, has argued that teaching at **all** levels is not solely a matter of learning the techniques (1986, 1987). This, he contends, has been the dominating approach in most teacher education programs. Instead of techniques, Shulman stresses the importance of teachers' first understanding their disciplines. As we

consider the socialization into the discipline of mathematics education, we must take in to account the different veins of knowledge for preparation in the field: mathematics education, education and mathematics. Learning about education and what it means to be a teacher of mathematics is important for future mathematics educators. However, it is also important to learn the discipline of mathematics and what it is to do mathematics in our preparation. The participants in the study voiced their desire for appropriate and meaningful opportunities to do just that. The courses identified as *novel* were opportunities to engage with mathematics in ways that felt meaningful and important to participants.

Experts in the field agree that mathematics education doctoral students should have mathematical experiences that are broad and deep (NCTM & AMTE, 2002). In this chapter, participants reported on *novel* experiences that appear to reflect the notion of “broad and deep“ in their courses which represented a way of learning mathematics that not only had more depth than most prior experiences, but also provided an opportunity to experience the discipline in way concurrent with the way a mathematician might engage with mathematics. The descriptions of the way authority was shared among student and instructors, the high level interaction, the nature of what and how mathematics was studied, and the sense of purpose in the course all impacted the participants’ experience.

The *novel* mathematics course experiences occurred infrequently, if at all, during participants’ doctoral coursework. In other words, the majority of mathematics coursework was reported by participants as being *typical* as reported in chapter six. One series of courses, *Foundations of Mathematics* (FM) came up repeatedly in participants’

discussions about *novel* course experiences. In the following analysis, I highlight these courses as a special case of *novel* courses.

Authority

The *novel* experiences that participants shared were most often ones that offer students a voice, establish an intellectual community of learners rather than a strong-arm knower-learner environment. These mathematical environments are needed to increase mathematical learning and meaning-making (NCTM, 1991, 2000; Cobb, 1989; Wilcox et al, 1991). Participants contrasted their *novel* experiences to the experiences they had in other mathematics classes where typically the, “professor and the textbook were the sole sources of authority”.

Cobb (1989) stresses the importance of encouraging classrooms where students and teachers work together as meaning-makers. In addition, he suggests that students should be allowed to do their own “truth-making”. Participants discussed their experiences in the *novel* courses as opportunities to share authority and work together-student and instructors. In their descriptions, it was clear that the instructors were not present as the sole source of expertise information or advice was. In addition, there was no “bible” or text of mathematical jurisdiction. Instead the authority seemed to reside within the members of the classroom community.

Authority in FM: A special case.

With particular respect to the FM courses, Kristen, Eva, and Anne Marie, shared the importance of the course project, which gave students the authority to pursue a topic or problem of interest on their own. Kristen shared, “It was just great to explore problems that we were developing on our own.” In addition, all three participants shared

being comfortable admitting things that they didn't understand in the FM course. They felt the shared authority in the class provided them with a space unlike the majority of the classes that they had taken in their programs. Kristen felt that this class afforded her the authority to say, "You know what? That didn't really make sense to me back then and I do have this misconception between my intuition and what I know to be true because it's in the math books." Anne Marie felt the shared authority, although critical, did take some time to get used to, as it was so different from other mathematics classes she had taken. Eva also mentioned an episode of uncomfortableness associated with being a learner in this type of environment with shared authority and loud student voice.

The participants' descriptions of the instructors' authority in FM illustrate what Philip Steedman (1991) distinctions in his description of between being "an authority" with "being authoritarian." In the FM classes, the instructors appeared to abandon the traditional role of a mathematics teacher as authoritarian. Steedman suggests that most mathematics teaching of the past has relied too much on the mathematics teacher as authoritarian. Letting go of the authoritarian role, Steedman continues, does not necessarily mean that the instructor abandons his or her authority. Rather, the authority is determined by expertise and maturity instead of the arbitrary experiences of power. This held true to the participants' descriptions of the importance of the instructors' expertise and authority in particular situations. All three participants commented on the shared authority in the class but also acknowledged the important role that the instructors took on. Kristen shared, "We looked to them (literally) when we had mathematical questions."

The experiences of Kristen, Eva, and Anne Marie suggest the Foundations of Mathematics course employed approaches that emphasize authentic, challenging projects that include students, teachers, and experts in the learning community. All of these, as suggested by Bransford, Brown, & Cocking (1999), are in accord with a constructivist framework. According to Bruner (1990), it is impractical in constructivist classrooms for teachers to make all the decisions and deposit information to students without involving students in the decision process.

Interaction

Recent research indicates the importance of mathematics classrooms in which students discuss their mathematics and explain and justify their solutions (Yackel, 2001) and that students learn through opportunities to explain, justify, and listen to the ideas of classmates (Maverich, 1999). Participants shared examples of 400 level mathematics classes where students discussed mathematics together and were afforded opportunities to explain, justify and listen.

These more interactive experiences did not, however, resemble a picture of a classroom where students and teacher were actually working together on mathematics, which is an even higher level of interaction. This, according to (Wilcox et al, 1991) is likely to engender in students a very different view of mathematics from the novel rule-and-procedure orientation. The interaction in these classes appeared to be for the purpose of encouraging students to communicate (“there was a lot more space for discussion/dialogue with all members in the room”) and engage on some higher level of problem solving.

Interaction in FM: A special case.

Kristen, Eva, and Anne Marie shared the importance of a high level of interaction in the FM courses. They collectively spoke about the collaboration, community, discourse, sharing and being shared to and the doing together of mathematics that was unlike most of their previous mathematical experiences.

Overall, Kristen, Eva, and Anne Marie shared similar ideas about the nature of the interaction within the course but emphasized different aspects of that interaction. The majority of Kristen's thoughts about the interaction were focused on the ability of the course to effectively reach students from variety mathematical backgrounds. For her, course was witness to the possibility of doing this well. It was this course that allowed her to experience a range of mathematical ability among peers from a learner's perspective and see the benefit from an educator's perspective.

The participants also spoke of the FM instructors learning alongside the students in the interaction that occurred. The course was a space that did not place emphasis on the individual identities of student and teacher. Rather it was a space that blurred those lines. Philosopher Martin Heidegger describes such an environment:

Teaching is even more difficult than learning. We know that; but we rarely think about it. And why is teaching more difficult than learning? Not because the teacher must have a larger store of information, and have it always ready. Teaching is more difficult than learning because what teaching calls for is this: to let learn. The real teacher, in fact, lets nothing else be learned than—learning... The teacher is far ahead of his apprentices in this alone, that he has still far more to learn than they—he has to learn to let them learn. The teacher must be capable of being more teachable than the apprentices.

In reference to philosopher Martin Heidegger's notion, Hultgren (1995) further expounds on "to let learn" as, "To prepare a space for listening that intertwines identities (self/other and self/society) in a retrieval of being, a leading in itself that withdraws from

teacher to being-in-teaching together (p. 377). In the FM courses, it was reported by the participants that very often the instructors were not merely teaching, but rather teacher and students were “being-in-teaching together”.

The discourse that occurred in the FM courses was described by the participants as consistent with the ways that researchers suggest as having positive influence on student’s learning and conceptual development in mathematics. Maverich (1999) suggests that students learn through opportunities to explain, justify, and listen to the ideas of classmates. In addition, Krummheuer (1995) indicates that participation in negotiations and argumentation has a positive influence on student’s conceptual development in mathematics.

The community of learners within the FM course, as described by the participants was critical to the courses experience. According to Wilcox et al (1991), the aspect of community within mathematics class has a powerful impact on empowering future teachers. They posit, that, “Community can be used to encourage the propensity to value mathematics, to use mathematics, and to share mathematics.” In addition the authors claim that collaborative help the members of the community develop several important propensities (a) toward service and sympathy- collaborative learning, (b) toward the suspension of impulse and the valuing of different methods and points of view, (c) toward listening to, understanding, and taking responsibility for one’s collaborators, and (d) toward the negotiation of what is to be allowed into the domain of working knowledge of group. (Wilcox et al, 1991 p. 4). Each of these was present in the discussions of the participants’ claims about the interaction and community within the Foundations of Mathematics classes.

Another key aspect of the FM course that has ties to a constructivist perspective is the fact that the environment allowed students to, “be led to talk about their thoughts, to each other, to the teacher, or to both. To verbalize what one is doing ensures that one is examining it. (von Glasserfeld, 1989, p.xviii).” All three participants discussed opportunities to share their thoughts and justify their mathematical work to other students and instructors.

Mathematics

Faith’s *novel* experiences, along with some of Anne Marie’s *novel* experiences, resembled Richard’s description of Journal Math, which is considered more like the language of mathematical publications and papers. The emphasis of Journal Math is on formal communication and reconstruction which is more in alignment with the heavy emphasis that was placed on proof. These approaches are reminiscent of Dewey’s description of “traditional” approaches that rely on “bodies of information and of skill that have been worked out in the past.”; therefore, fulfilling the main purpose of schools of transmitting that knowledge to the new generation (1938, p. 17).

Mathematics in FM: A special case.

Doing mathematics via inquiry, thinking, or “having one’s mind aroused” as the etymological roots illustrate, was the way in which Kristen, Eva and Anne Marie’s consistently described their experiences in the Foundations of Mathematics courses. Each participant mentioned the nature of mathematical content, how mathematics was discussed, and whose mathematical ideas and questions were important in the class. There appeared to be curiosity in the kind of mathematics that students were afforded the

opportunity to engage with. The ideas came from the students; the mathematics came from working together and from students' experiences and questions.

As current and future teachers of mathematics and mathematics education, it is important for mathematics education doctoral students to experience depth and vastness in mathematics learning that expert in the field suggest. This learning and understanding has been described as "broad", "deep", and "thorough" (Ma, 1991; Ball, 1989; NCTM & AMTE, 2002). Ma (1999) suggests, "breadth could be thought of as the widely different spheres of experience that can be related to one another; depth can be thought of as the many different kinds of connections that can be made among different facets of our experience (pg. 121)." Ma refers to Duckworth (1987, 1991), a former student and colleague of Jean Piaget, who suggests that breadth and depth "is a matter of making connections," and that the two are interwoven (pg. 121). All the participants shared the importance of mathematical connections that students made in the course and how the mathematical knowledge of students provided a breadth and depth of resources from which to draw upon.

Wilcox, Schram, Lappan, and Lanier (1991) speak of the importance of doing mathematics in a way that instills a sense of discovery in students' learning as a way to impact teachers' mathematical knowledge and beliefs. "The fact that others have previously passed this way mathematically does not take away from the excitement and the struggle that can accompany personal and group sense making in mathematics (p. 3)." Having a sense of mathematical discovery emerged as being an important part of the FM course experience for Kristen, Eva, and Anne Marie.

Purpose

The theme of purpose emerged in *novel* courses as being important to participants for a variety of reasons. For some, the purpose of taking the class was to gain a sense of how one would actually go about investigating math, to provide an example of how things in a mathematics class could be different from the *typical*, or as an opportunity to do mathematics in a way that students had learned about but had not had opportunities to do. For others, the purpose of the mathematics in the *novel* course was to provide an avenue to experience the joy of posing and solving questions that matter to students or to explore problems that students were developing on their own.

Purpose in FM: A special case.

Among the three participants who took the Foundations of Mathematics course, Kristen and Eva mentioned taking the courses because they needed to take them. Both Eva and Anne Marie mentioned that they were more willing to register for the courses because they “knew the instructors” and felt that they would be supported as learners in the course. According to the syllabi, the goals of the course were as follows:

For Kristen, the course was an avenue to experience the joy of posing and solving questions that matter to us”, “seeing it [the kinds of mathematics that mathematics education students learn about in their education classes] in action”, and opportunities to “explore problems that we were developing on our own.” The class provided Kristen an example of how things in a mathematics class, “could be so different” [from novel mathematics courses].

For Anne Marie, the FM courses were less an experience of testimony that this kind of mathematics teaching and learning could occur, but rather an opportunity to do mathematics in a way that she had learned about but had not had opportunities to do. It was a course that allowed her the freedom to feel valued mathematically. It was a place where she felt that her questions and interests mattered, and her not-knowing wasn't a character flaw to be fixed rather a strength to be acted upon.

Learning environments are important elements of instruction, but students cannot be thrown into complex settings and left to themselves. They need the kind of help that comes from interaction with peers, with a knowledgeable teacher, and with relevant tools and information. There is thus a continuing dialectic or tension between student-initiated exploration and teacher guidance. (Wilson et al, 1993, p 2.)

The participants' reflections present a picture of a mathematics learning environment consistent with many pictures of constructivist classrooms that experts in the field are pushing for in providing rich and meaningful mathematical experiences. The shared authority and high interaction environment in FM encouraged and valued the exploration of problem situations. It was an environment of modeling, justification, exploration and argumentation. Kristen, Eva, and Anne Marie reported on the support received from peers and instructors as well as the role of the instructors as critical to their experiences.

Overall, the participants' recollections of their experiences in the Foundations of Mathematics courses were truly unique to the mathematical experiences of the overall pool of participants in this study. Their experiences were unlike most any other mathematical experiences they recalled having in the past and made a strong and positive impression on the participants thinking about the teaching and learning of mathematics.

These experiences, according to participants, were critical in their overall preparation in becoming mathematics educators.

Conclusion

This chapter reported on participants' descriptions of their mathematical experiences in courses classified as *novel* by the researcher. These initial descriptions were first presented through the use of the phenomenological tool of "bracketing" (van Manen, 1990). The majority of *novel* course experiences that participants reported on described a course scenario where students and instructors worked together, students' voice and questions were crucial to the content and direction of the course, the mathematics was playful, meaningful and from the students, and students felt purpose in the work they did and the mathematics they learned.

These descriptions were followed by experiential descriptions of these experiences including the meaning of these experiences for participants. "Experiential descriptions" uncovered participants' emotions, perspectives, preferences, expectations, and feelings about their experiences. Experiencing mathematics in this unique way was influential, even transformational, for mathematics education doctoral students in this study.

Although, this chapter highlighted unique, or *novel*, course experiences and participants' reactions to those experiences, it is important to revisit the fact that these *novel* course experiences were not the kinds of mathematical experiences that mathematics education doctoral students in this study typically had. The overwhelming majority of mathematical course experiences reported on by participants in this study would be classified as *typical*. This reality of contrasting course experiences provides the

backdrop for rich discussion among participants. Next, a vignette is provided that highlights participants' reflections on the contrast that exists between the kinds of the mathematics courses typically experienced with the "Utopian" experiences that a mathematics education doctoral student, future steward of the field, would desire. Chapter seven follows the vignette. In chapter seven, I will summarize the research, provide an analysis of the data, suggest limitations of the study and propose avenues of future research. Following chapter seven is Mathematical Interlude Three.

VIGNETTE

Only after we have looked at something, not as it 'is' but as it is turned inside out or upside down, do we see its essence or significance (Brown & Walter, 2005, pg. 30)

Do we always "see" what is in front of us? The most obvious things are frequently those most hidden from us. Sometimes it takes a bit of a rude awakening for us to appreciate what is right before us and, often, some kind of re-organization or shift of perspective leads us to see the obvious (Brown & Walter, 2005, pg. 30)

In mathematics, our ability to 'see' is very often defined by our ability to solve. In an attempt to provide students with clearer vision, mathematics educators Stephen Brown and Marion Walter presented a problem solving strategy in their book, *The Art of Problem Posing* (1983). The strategy, a mindset of tackling problems, encourages students to consider the art of problem posing in relationship to problem solving. Brown and Walter's strategy helps students solve problems through posing new problems in the very process of solving it via asking a series of "What-If-Not" questions. For example, if traditional strategies are not allowing a student to successfully use the Pythagorean Theorem in a problem situation, the student could think of the attributes of the theorem to pose new problems. Brown and Walter offer the following "What-if-Not" thinking to such situation: "How else could we construe the statement? What if it were not a theorem? What could it be? (p. 48)?"

It is the traditional, static thinking about mathematical problems that is very often hindered by taking the "given" for granted. In doing this, Brown and Walter posit, we watch our students become enculturated to the assumption that we must take the "given" for granted and thus become entranced by mathematical proof. Brown and Walter argue that there is more to mathematics than proving things. To that end, my dissertation

journey was and continues to be one of understanding and uncovering mathematical experience, not one of proving.

As I approached the central question of my study, “What are the mathematical experiences of mathematics education doctoral students?” I started my participant conversations with the obvious prompt, the “given”: *Tell me about your mathematical experiences in your doctoral program.* These results were foundational yet not exhaustive. Therefore, in a subsequent group conversation, rather than asking questions from a traditional channel, I engaged participants on a “What if Not” journey. What if we were not mathematics education doctoral students? How could we go beyond accepting the “given”? What if we stepped outside of the “given” that mathematical experiences in doctoral programs appear to be rigid, consistent, and unchangeable? The following conversation is the fruitful response to our willingness to step outside of the given and seek to understand the significance of our experiences:

Anne Marie: We are going on a trip today. You will need to be packing your suitcases with all the essential items that will help you once we get to our destination- things that you don't think you should be without. Our destination today is to Utopia University. We have all been hired as new mathematics education faculty and will be starting tomorrow. What needs to be in your suitcase? (regardless of whether it is in your closet now) For example, what knowledge, experiences and dispositions will be most important for you to bring? Of those items you need to pack, which do you feel are in your closet as of today? Which are not? How does the level of your preparedness for this trip make you feel?

After we spent time writing about and sharing the contents of our closets and suitcases, I continued:

We are still at Utopia University. We have been teaching there for several years and everything is wonderful. We have had the opportunity to establish the ideal doctoral program in mathematic education. It has been going on for about 5 years and our first cohort of students are getting ready to graduate. We want to take stock and ask our doctoral students about their experiences, specifically their mathematical experiences in the program. I want you to write down the kinds of descriptions you would want to hear from our mathematics education doctoral students. What kind of things would you want

our students to comment on in regards to the mathematics classes they took? What were they like? How did they make them feel? What you want them to say about their role in the classes. What typically was the teacher's role? If we were to interview them and say, "Tell me about your mathematical experiences- what were your math classes like?" If we had done the best job at Utopia U, what would they say?

Eva: [I would want my mathematics education doctoral students to say:] *I am really "doing" math. Even if I don't know the math right away or how to answer a problem, I have the confidence that I could work through the problem. Working with others really helped me be more creative and opened my eyes to different perspectives and problems in math. I now feel that mathematics is really creative and that asking questions is a part of doing math. I always looked forward to coming to class and thinking about mathematical ideas. I was totally exhausted by it because I always wanted to think about the math.*

Mia: [I would want my mathematics education doctoral students to say:] *I understand math! Math makes sense! My math experience has been like the thing that I learn about in elementary math methods. I see how this helps me as a math teacher. I actively participated in developing my understanding of math - it wasn't just someone else's knowledge.*

Faith: [I would want my mathematics education doctoral students to say:] *My math experiences were so meaningful and engaging. I can remember what we talked about in such-and-such class. I'm glad we got to take semester long courses thinking about and examining each major content area and how that evolves throughout the grade bands. I love the fact that we had the opportunity to discuss and closely examine mathematical ideas as they unfold for a young child and see how it connects to a higher-grade level.*

Wendy: [I would want my mathematics education doctoral students to say:] *All the math classes I took were taught conceptually and meaningfully as opposed to traditionally. I revisited the math that I plan to teach and I revisited on a much deeper level. Some of the math was above my math but I was able to see connections between my level and the other.*

Kristen: [I would want my mathematics education doctoral students to say:] *Math was fun and I looked forward to going to class. It was empowering in the sense that I felt like I was important in the development of ideas. My classes helped to connect concepts and ideas, it was full of a-ha moments, and it was logical and applicable to k-12 teaching. It was useful and it helped me to revisit and think more deeply about "simple" mathematical ideas. It was student centered and a model for how I want to teach.*

Anne Marie: [I would want my mathematics education doctoral students to say:] *We worked together- both teacher and students. We made connections. We did a lot of projects. The teacher didn't lecture the whole time. Math made sense. It was hard. There was a lot of thinking but it was meaningful. I saw the k-12 math connections. I learned a lot!*

Prior to our Journey to Utopia, I had the opportunity to engage in individual conversations with my fellow travelers about their actual mathematical experiences in the mathematics education doctoral program that we had been enrolled in. It was these real experiences that I wanted to learn more about and so I continued our conversation by using our Utopian ideas to “see” our lived experiences more clearly.

Anne Marie: *I pulled out from your earlier conversations with me the words that came up most frequently when I asked you to describe what your actual mathematics classes were like in your doctoral program (and in the courses that were prerequisites to the mathematics in your doctoral program). I've taken note in the places where you said, “this is really great” or “this class was really great” and I will continue to ask about those isolated experiences. But the following list includes the overwhelming majority of how you described your math classes. These are the words that you used. I'm going to read them and after I'm done reading them I want you to write down your feelings about what you heard. I want you to consider the similarities and differences between the lists that you generated about Utopia University and those you have share with me about your own experiences.*

I continued by reading from the list:

‘Talking to the board. I had no idea what was going on. Lecture. Follow the steps. He was not able to explain the math. They didn't know how to teach. We didn't ask questions. There were no projects. We didn't work together. I was miserable. It was humbling. There were a lot of problems. The tests were impossible. I don't understand. It was frustrating. There were a lot of handouts. They knew “it” but they couldn't teach “it”. Textbooks. Failing tests. Do I really care? Feeling stupid. Incompetent. Petrified. I just go through it. It was a means to an end. Why do I have to take this? Proofs. It was confusing. It was never meaningful. Why am I here? I don't remember the math. I don't remember the class. Bringing sheets of formulas into the final. Professors talking about what he wanted to talk about. We worked individually. It was the worst class I ever took in my entire life. Blah blah blah. There was no excitement. It felt purposeless.’

After several minutes, we shared our reactions to the descriptions.

Faith: *It was completely opposite to what we all said that we wanted at Utopia. And then I had some other reactions that came just off the cuff. I asked, “How is it possible to develop our Utopia University experiences without the right instructors packing our own suitcases?” Can those of you who have experienced “it” teach a class like that? I mean you all have seen it but I know you all have talked about its one thing to sit in it versus do it yourself. Why bother to change what is often the status quo of the way we were all taught if the powers that be perceive that that kind of math is the big M, like the kind that Mia was talking about, and not the kind of math that was experienced by some of us. I*

just feel like, there's that issue of what's been happening for years and years and years and ...

Anne Marie: Exactly. First, I was sad. Just because it is so opposite and its so obvious and overwhelming. Partly because I know the other side. Like, I got a little taste of Utopia I think. And not to blow it out of proportion but a lot of what we talked about – what was on that list, Foundations of Mathematics was the first time I experienced those things (The Foundations of Mathematics courses mentioned were a set of courses that several of the participants had taken during their program of study). Next, pissed off that I'm expected to preach that this is how you should do it to my undergrads that nobody ever learns like that. So I guess I'm pissed that there aren't more opportunities. Also scared because I don't know how to teach that way. I'm not an experienced learner of that- Utopia...so how does that transform my teaching? I'm not going to be able to teach that class. I'm not going to be able to do what our instructors in Foundations of Mathematics did. So, I'm a little scared. I'm a little bit hopeless...But I also feel empowered because I know its possible to have really positive learning experiences in mathematics beyond elementary school. And I want to help change that hence sitting around this table. So those are my words. And is one or two courses or experiences enough? Is five enough? Are five semesters of thirty years going to school enough to understand what that means? I don't know.

Kristen: I was also sad that we'd all had such bad experiences but I was also surprised that we are all so dedicated to mathematics education. It's been so horrible for many of us and what is it that's made us still love math and still want to spend our lives devoted to it? I don't know. If it's been so horrible, why are we still so interested in it? And I wasn't surprised by our descriptions.

Mia: You know, when you read the list of things that we had said earlier, you almost kind of transport yourself back into those classrooms and you start to feel like that frustration again and confusion. So those are the first two things that came out...I'm like oh, my gosh I'm back in those classes again. Like I can envision it.

Anne Marie: Sorry...

Mia: It's okay. And kind of this idea that math equates to something that is bad, it's not fun, it's just like a random collection of stuff. I said it's sad that what so many students experience in math is so different from what we wrote down as our ideal math classroom or environment. This is related to what Wendy said about its no wonder that so many people say they hate math or they're not good at math because these are the experiences they have.

Kristen: Related to that, why do we think math is so important though? If we've had such horrible experience...why do we still feel like it's important for kids to have better experiences.

Eva: Did any of us, did you guys love it?

Faith: *When I was little.*

Wendy: *I was good at it so I did it.*

Anne Marie: *It made sense for a little while until high school.*

Faith: *I remember when I was in high school being able to derive things. I didn't memorize. That was exciting. I got it.*

Eva: *I didn't have words like that although you'll notice when I read, I went on an angry or sad rage. So, I said it just feels like the way most classes are taught takes the heart and interest out of math and to what end? So, students leave math and never need it anyway. Why can't math be as fascinating, challenging, enjoyable as our Foundation of Mathematics experience? As our experience suggests, it seems so counterproductive. It robs kids, teachers, and the discipline when we teach math traditionally by those who aren't actually interested in teaching....I've been thinking for sort of a long time. I mean I sort of try to do some inquiry with the teachers that I work with and it's hard. And every time I look at my daughter... you know, the little ones. They ask questions all the time. The thing that I just keep thinking is we kill it. We kill it, we kill it, we kill it. 5th and 6th graders would be curious the way we're curious if we didn't kill it. We need somehow to get to the elementary teachers.*

Anne Marie: *To take it even a step further. It's been killed in us or at least silenced for so long about asking my own questions or feeling like I should ask my own question. So now, translating that into now having to go and tell a bunch of other people that that's what they are going to do. It hasn't happened for so long that we don't know how to do it. It's just a big viscous cycle.*

Faith: *I have to agree with you in that if we are going to make a change in mathematics education across the board, it has got to start with those who touch our young babies and that would be our elementary teachers.*

Anne Marie: *Those are the people **we** teach.*

Faith: *Yes! Those are the people **we** are responsible for!*

This conversation was a prominent reminder of why my research questions and study were important. It was in the screamingly obvious differences of what we (soon to be stewards of the field) claim to be important mathematical experiences to have as mathematics education doctoral students and those experiences that we actually had, that begs investigation.

CHAPTER SEVEN

Conclusions

This chapter provides the reader with an overview of the study, a set of implications drawn from the results, and a framework for thinking about the differences in the mathematics course-taking experiences of mathematics education doctoral students. Then, a list of limitations, as well as suggestions for future research, are provided. The chapter ends with final remarks from the researcher.

I was inspired to carry out this research after having the opportunity to engage with mathematics in a way that was unlike any other I had previously been afforded. These experiences occurred in a pair of courses, *Foundations of Mathematics (FM) I* and *II*, taken in my mathematics education doctoral program. The courses focused on student questions, problem solving, conceptual understanding, and shared authority. The course goals, according to the course descriptions, focused on providing mathematics education doctoral students with opportunities to “explore their own areas of mathematical interests from the areas of the curriculum relevant to their professional goals”, in addition to “offering illustrations of strategies for exploring mathematics, resources that focus on the mathematics of the K-12 curriculum, and reflections on the nature of mathematical activity and exploration”, and for developing “strategies that support lifelong learning of mathematics, particularly of mathematics related to their professional work.”

The focus of the FM courses was a shift from the primarily direct, rule-driven, telling-and learning instructional approaches that I had most typically experienced in mathematics courses. The FM courses impacted my understandings of mathematics

teaching and learning, as well as my mathematical ability and voice; they, thus prompted me to consider how my experiences in FM were situated within my mathematics education preparation. Literature pertaining to the nature of mathematics teaching and learning, mathematics education reform, and doctoral student preparation provided the framework for this study. I set out to understand the following questions:

3. What is the nature of the mathematical experiences that mathematics education doctoral students identify as components of their program?
 - What is the meaning of these experiences in the context of our doctoral student careers?
 - How are these experiences situated within our preparedness as we enter the mathematics education field?
4. How can specific mathematical experiences influence the identity and preparation of a mathematics education doctoral student?

In pursuit of understanding the aforementioned questions, I assembled a sample of six mathematics education doctoral students enrolled at a major mathematics education teaching and learning center. The sample consisted of five female students, all in advanced stages of doctoral study, and myself. As a fellow mathematics education doctoral student, my experiences and reflections added to the pool of data. Collectively, the participants had a rich and varied background of mathematics courses in their academic histories. Borrowing from both case study and phenomenological traditions, I asked participants to share their experiences and perceptions regarding the mathematical components of their preparation. This sharing occurred during three stages of conversations- both individual conversations with the researcher and group

conversations.

Initial conversations provided data regarding the participants' background and their stories of coming to the doctoral program. These conversations also provided me with a sense of participants' mathematical pedigree and mathematical experiences in their programs. Subsequent conversations produced richer data about the participants' experiences, including academic and professional expectations and needs in their mathematical preparation.

During one group conversation, I engaged participants in a What-If-Not exercise (see Brown and Walter, 1983) as means of understanding what advanced mathematics education doctoral students feel is essential in mathematics education doctoral programs and what it is these participants felt they learned from their own mathematics courses. We further explored these issues through conversations about working at a fictitious university. The conversations provided me with key experiential data regarding participants' mathematical expectations and desires as contrasted with those experiences actualized within their programs.

Additional conversations, email, and dialogue along with other data sources such as course materials, videotape, personal journals, and written course reflections provided a rich set of data. Common themes and patterns emerged from these data sources.

In my analysis of the abovementioned data, I identified several aspects of mathematics course experiences reported by mathematics education doctoral students. I determined that these aspects fell into four broad categories, which emerged as themes across the data. These themes were authority, interaction, mathematics, and purpose. Regardless of the level of courses that participants were talking about, these themes were

highlighted in the conversations about mathematical experiences in participants doctoral programs. Although aspects of all four themes impacted the mathematical experiences of participants in their courses, differences in the ways participants experienced these themes were evident.

The experiences of mathematics education doctoral students also varied in their mathematical coursework. I sought patterns and relationships among the themes; in doing so, these patterns and relationships fell into the categories of *typical* and *novel*. *Typical* course experiences were those that mathematics education doctoral students had come to expect as routine, often described as traditional. *Novel* course experiences were those that mathematics education doctoral students reported as being unique and reform-like.

Although most course experiences that participants spoke about were considered traditional, or *typical*, it was the courses that were described otherwise that stood out in the data. The courses that were referred to as “not traditional”, or *novel*, were primarily considered that way because of a shift in the authority in the classroom and/or a high level of interaction between the instructor and the students as well as among students. The shift in authority usually denoted a course where the instructor allowed students to make certain decisions regarding coursework or grading. With regards to authority, participants reported classes as being less traditional when students were allowed to work together in groups. In addition, the nature of mathematics and the sense of purpose were also distinct in these courses.

During conversations about their mathematical experiences, participants became very vocal and emotional about their experiences. Their expectations that mathematics courses would be traditional, *typical*, or “more of the same” was evident. Participants,

who had experienced *novel* mathematics courses, were excited and relieved that there had been an opportunity to learn mathematics in that way. There was a sense of disappointment in the pool of participants who had not experienced mathematics in that *novel* way.

As I consider the distinct differences in course experiences and the emotional reactions to these differences, I am drawn to the work of Stephen Brown. Brown, one of the founding fathers of the Humanistic Mathematics Education (HME) framework, has articulated a description of a framework that has helped me conceptualize the results of the study. The HME framework represents a paradigm shift from traditional models of mathematics teaching and learning. It has provided a particular way to think about experiencing mathematics.

Humanistic Mathematics Education: A Framework for Mathematical Experiences in Mathematics Education Doctoral Programs

Dossy and Lappan (2001) state that the important thing to consider in the mathematical education of mathematics educators in doctoral programs in mathematics education is that individual students be able to “do mathematics” at an appropriate level. To “do mathematics” at an appropriate level for a doctoral student in mathematics education”, according to Dossey and Lappan, “means that the student is able to:

- Appreciate the rules of evidence within the discipline;
- Outline and connect the major ideas within the discipline;
- Analyze and apply the major algorithms and procedures within the discipline;
- Describe the ways of thinking through which the discipline itself expands;
- Use disciplinary knowledge to solve problems in the discipline and related

disciplines; and

- See the connections among and between ideas, concepts, structures, and methods within the discipline and outside the discipline (pg 67).”

There is no doubt that most mathematicians and mathematics educators would concur on the importance of this type of mathematical knowledge. These are indeed important things that mathematics educators will need to know how to “do” in their professional work. The results of this study have convinced me that doctoral students need something more, in addition to the above list, in their mathematical education as doctoral students.

Most of the mathematical experiences as reported by participants in this study have resembled Browns’ description of experiencing mathematics as a depersonalized, uncontextualized, non-controversial and asocial form of knowledge. Having experiences that depart from this characterization in our doctoral preparation have been considered so distinctive that participants have called them unique, amazing, and even transformational. The nature of these unusual experiences appear to fit the HME framework which represents a reaction to this depersonalized and uncontextualized view of mathematics teaching and learning.

The framework was a collaborative effort. In 1986, a group of university mathematicians, mathematics educators and philosophers assembled to, “discuss the relationship between mathematics and the humanities, and more generally to discover what was wrong with how the discipline of mathematics was being portrayed to its clients at elementary, secondary and university levels” (Brown, 1996). Brown and others acknowledged there was something “wrong” with the way mathematics was being portrayed to students at all levels- even the university. The group, created a list of tenets

that help define the work surrounding Humanistic Mathematics Education:

- a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be 'merely technical'.
- b) An appreciation for the human dimensions that motivate discovery -- competition, cooperation, the urge for holistic pictures.
- c) An understanding of the value judgments implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated and *why* it is investigated.
- d) ...a need for teaching. learning formats that will help wean our students from a view of knowledge as certain, to-be-received.
- e) The opportunity for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues.
- f) Opportunity for faculty to do research on issues relating to teaching, and to be respected for that area of research.

There are two different, yet related, aspects of HME: They are: (1) teaching humanistic mathematics (as in teaching a view of mathematics as a meaningful human enterprise sharing many of the assumptions of other humanistic studies and experiences) and (2) teaching mathematics humanistically (as in treating students with dignity and respect and concern for their awareness) (Brown, 1996). The first, teaching humanistic mathematics, refers to a transformed vision of the nature of mathematics. It suggests that one would teach an alternative to the view of mathematics being absolute and certain: Students would be taught in a way that portrayed mathematics as imperfect, messy, fallible. This includes a sense in which both mathematical meaning and proof are socially constructed. The second orientation of humanistic mathematics, teaching mathematics humanistically focuses on the learner rather than the discipline. It requires a commitment to the interests of learners and to the way in which they acquire meaning. (Brown, 1996).

Brown provides the following matrix to consider how these two aspects are connected.

		THE LEARNER	
		learning as received knowledge	learning as constructed knowledge
NATURE OF MATHEMATICS	math as absolute	A	C
	math as fallible	B	D

A program that is lacking in both humanistic qualities is depicted in box (A). These could be characteristics of the kinds of experiences that all participants described as most typical in their programs. The way in which we most typically experienced mathematics was as receiving knowledge as if we were “devoid of human agency” (Brown). The consistent efforts to hide one’s inner thoughts leads to the belief that there is very little meaning associated with the mathematics we do. There were no course experiences that participants reported on that might fit into box (B), which describes a program where students might be expected to be “receivers of knowledge” yet in an environment where mathematics is seen as imperfect and fallible.

Brown posits that a “full-fledged humanistic program [incorporating both (1) and (2)] is depicted in box (D)”. The data in this study suggest that the *novel* course experiences as reported by participants, and in particular those emerging from the *Foundations of Mathematics* courses, appear to fit best in box D. Participants reported

constructing their own understanding through their interaction with each other and the mathematics. In addition, the environment was such that participants came to see mathematics as messy and fallible, experienced the struggle with challenging problems as well as how to make progress with a mathematical problem, learned how to ask questions and what to do with dead ends. Arguably, these are all opportunities that mathematics education doctoral students should experience in order to form a broad understanding of mathematics.

Despite (a) its constructivist orientation, (b) its focus on interest of students, (c) its awareness of the various roles of the social context of education, (d) its challenge to the fallible nature of mathematics, (e) its awareness of the potential of history to illuminate the human agency in mathematics, there has been a legacy from an earlier age that dominates much of curriculum. That is the field is still dominated by a rather narrow view of the purpose and uses of problem in mathematics (Brown, 1996).

It is this legacy, I posit, that has influenced the dichotomy in experiences in mathematics education doctoral preparation as reported in this study. In order to best prepare mathematics education doctoral students for their future roles, it behooves programs to offer students mathematical course experiences that provide a view of mathematics that is much wider and more human in its purposes and uses. This may require programs and policy makers to examine the history of the role of mathematics in the mathematics education profession and consider how mathematical experiences have helped to prepare or not prepare doctoral students for becoming Stewards of the Discipline. If doctoral education serves as socialization for the profession, an examination of the mathematical preparation process is appropriate and necessary given the mathematical breadth and depth these Stewards will face in their careers.

What I Learned

Mathematics education doctoral students enter doctoral programs with a myriad of backgrounds and mathematical histories. They bring a depth and breadth of knowledge and experience. Doctoral programs are largely responsible for meeting these students at the door and providing the coursework and experiences necessary for helping students develop into Stewards of the Discipline. Included in this charge is to help these future Stewards develop broad and deep mathematical knowledge that will help them in their professional roles as Stewards.

As educators, the participants in this study were passionate about their perceptions of what the nature of the teaching and learning mathematics should be. They were also concerned about the way in which students, writ large, are affected by various modes of mathematics instruction and pedagogical decisions in mathematics classrooms. The participants, students in advanced study of mathematics education, had a substantial corpus of knowledge and experiences in teaching and learning, which provided important and valid insights. The participants expressed disappointment as they came to expect their mathematical experiences as rarely, if ever, being about their own mathematical interests and questions. Participants further recalled their desire of wanting courses in their doctoral programs that would be meaningful, help them make connections and be classified as *novel*.

As students in mathematics education, participants were well aware of the research that supports the idea that mathematics teaching needs to undergo change in the United States. The common traditional mathematics classrooms have been described as having, “orderly chairs, straight rows, and teachers in charge” (Lindquist & Elliott, 1996)

which the mathematics education doctoral students are still most accustomed to in their mathematics classes.

Mathematics education doctoral students, Future Stewards of the Discipline of mathematics education were also aware of the vision of the reform mathematics classrooms, where students are able to experience “problem-based learning, inquiry activities, dialogues with peers and teachers that encourage making sense of the subject matter” (Windschitl, 1999, p. 1). Yet, it was not surprising to learn that the overwhelming majority of the participants’ mathematical experiences were reported as *typical*. There was a taken-for-granted attitude from participants that most mathematics courses in their programs were this way. This mismatch between understandings and experiences strongly suggests the need to examine what it means for mathematics educators to know mathematics in their professional identity, what mathematics is important to uphold the integrity of the field, and how best to prepare doctoral students in light of both of these issues.

One particular set of courses deemed “influential”, “transformational”, and “empowering” embodied a classroom where inquiry activities predominated and the course environment allowed, required, and supported students to explore their own mathematical questions. This set of courses also featured a shared authority between instructors and students, engaged students in a high level of interaction, and explored mathematical questions and problems that came from the students’ questions and ideas rather than from a textbook. The purpose of the courses was a place where students could have an authentic opportunity to *do* mathematics. The significance of the mathematical inquiry activities in these courses, according to participants, was not only important for

themselves as learners of mathematics but also critical to their becoming mathematics educators. According to the participants, these experiences provided a model of mathematics teaching and learning that fulfilled a missing professional need that they had been seeking.

I see these *novel* mathematics courses as being critical for the development of a Steward of Mathematics Education. *Novel* experiences must be a part of doctoral student preparation in mathematics education. These experiences significantly contribute to doctoral students mathematical knowledge, add to their understanding of the models of teaching and learning that exist, and provide opportunities to experience mathematics in ways concurrent with what students learn in other coursework within their programs of study.

This research has serious implications for doctoral student preparation. It illustrates the clear differences of what soon to be Stewards of the Discipline claim to be important mathematical experiences and those experiences that they actually have. The participants very often referred to their professional preparation in their discussions and many of them shared concern that they did not get the “deep meaningful k-12 connections” that they had hoped. Others shared the importance of taking the FM courses as having a model of how this kind of experiences is necessary as a model to draw upon. With respect to the effect of these experiences on mathematics education doctoral students’ perceptions of their preparation, the students who had taken the Foundations of Mathematics course sequence expressed their insight into the important ways it affected their preparation.

In a chapter on the mathematical education of mathematics educators, Dossey &

Lappan (2001) acknowledge that the creation of “new, carefully constructed programs of study” are needed for adequately preparing doctoral students. They posit “Meeting the needs of mathematics educators will require a transformation of the undergraduate and graduate programs for what might be considered solid preparation in mathematics for candidates in mathematics education.”

Dossey and Lappan’s statement indicates that current preparation in mathematics for candidates in mathematics education programs has not been adequate. The results of this study are concurrent with their acknowledgement. The call for a transformation in mathematical preparation is lofty indeed. The disappointment, anger, and frustration that emerged in participants consideration of their mathematical experiences within their preparation is indication for heeding the call for further consideration of the mathematics preparation of candidates in mathematics education. I recommend that developers of mathematics education doctoral programs seriously consider the results of this study in light of the calls by Dossey and Lappan, Reys, NCTM, and others in their design of programs.

Earning significant mathematics credit has been part of the preparation of doctoral students, as well as a credential for becoming a Steward of the Discipline. The history of the field and the nature of the relationship between the field of mathematics and mathematics education may both be influences on why mathematics education doctoral students are advised and required to “take more math”. For the participants in this study, however, taking more *typical* courses did not meet their personal and professional mathematical needs. The classes, from the participants’ perspective did not broaden and deepen their mathematical knowledge. This study suggests paying closer attention to the

mathematical experiences provided within coursework rather than mere number of credits taken.

Meeting the mathematical needs of mathematics education doctoral students is critical in preparing them for being coming Stewards of the Discipline as Dossy and Lappan (2001) suggest. Assuming these needs will be met through the process of students taking more 400 level, very often *typical* courses, as this study highlights, is insufficient. Taking mathematics courses as part of a mathematics education doctoral program should not be considered another hurdle or requirement to meet in preparation but rather, an important part of doctoral students' understanding of mathematics for becoming Stewards. For the participants in this study, *typical* graduate level mathematics classes were most often viewed as hurdles. These hurdles, beyond not fulfilling the participants' mathematical needs and expectations, were often disappointing and even offensive.

It is my contention that such personal and visceral reactions to mathematical experiences stems from the fact that the participants in the study came to their doctoral program with rich histories and experiences that had deeply shaped their understandings and expectations of what mathematics education should be. These understandings and experiences were repeatedly confronted and challenged with the *typical* course experiences in their mathematics classes. After spending years of deepening their own understandings of mathematics teaching and learning, as well as improving their own practice of teaching mathematics, participants chose to continue their careers with the goal of becoming Stewards only to face mathematical hurdles that not only, very much contrasted their own understanding and beliefs of what mathematics teaching and learning should be, but also weren't seen as particularly useful for their own goals of

becoming Stewards of the Discipline. If the field of mathematics education has aspirations of improving the direction and future of the field, the issue of preparing its Stewards becomes critical.

I deliberately chose to include my own mathematical experiences in this study not only to provide another doctoral student experience to the data set, but also because I believe that there can be something to be learned from my unique journey towards Stewardship. My experience provides voice to those who come to mathematics education doctoral programs as former elementary school teachers, perhaps those with less extensive mathematics preparation but nonetheless with important and vital knowledge and experiences of K-12 mathematics teaching and learning.

If the only students who are accepted into mathematics education doctoral programs are those with a corpus of particular mathematical experiences, the pool of potential mathematics educators is greatly narrowed. Further, if the only doctoral students that successfully meet the mathematical requirements in order to graduate from mathematics education doctoral programs, the pool of potential mathematics educators is further diminished. This weeding out of students could potentially eliminate many classroom teachers, curriculum specialists, and other professionals in the field, as well as those potential candidates who are eager and capable but limited in their mathematical promise and achievements because of the nature of mathematics courses and support they had been afforded throughout their careers. In this scenario, the discipline of mathematics education would be confined to a group of Stewards whose background, experience, and credibility have been focused primarily on mathematics. This results with a pool of Stewards, who are equipped with a very particular set of experiences, backgrounds, and

understandings of mathematics. Therefore, in order to ensure a field of Stewards who offer a breadth and depth of knowledge, it behooves the field to seriously consider the programs, specifically the mathematical preparation, of its future Stewards. I posit that providing students with more *novel* course experiences, as described in this study, would be one way to help better prepare mathematics education doctoral students for their future roles as Stewards of the Discipline.

Stewards of Mathematics Education enter the field with a wide range of professional opportunities awaiting. These opportunities require a wide range of requirements and professional expectations. One important condition that permeates all of these professional opportunities is to know mathematics. Mathematics is the gem in the work that we do. As a discipline, mathematics has grown out of centuries of asking questions, probing, exploration, invention, frustration, and being delighted. If Stewards are charged with the passing knowledge on to others, it behooves mathematics education doctoral programs to provide Stewards with more opportunities to experience and understand mathematics as a discipline.

Limitations

It is important to recognize that there were some limitations to this study. Most of these limitations are a result of premeditated methodological choices made by the researcher. In keeping with the qualitative nature of the study and wanting to gather rich personal data, I choose to engage with a small sample of participants. However, by interviewing a small number of mathematics education doctoral students enrolled in a particular program ($n = 6$), this project is limited in its breadth. Further research could provide broader results.

Another limitation of this study is the result of the fact that the insights, experiences, and reflections provided are from the individual perspectives of mathematics education doctoral students. For the purposes of this study, the participants were doctoral students who were in advanced stages of study. That is, I did not interview faculty, policy makers, or other individuals who hold a stake in or influence on the mathematics education doctoral student experience. Faculty input and reactions to the mathematical experiences of doctoral students are a helpful piece of the puzzle, but they are not critical for determining the validity of the arguments made in this study.

Another limitation of the study is a result of the subjective nature of the data analysis process. The categorizations that came about as a result of the analysis were developed by the researcher through investigating emerging patterns among participants' responses. A description of the criteria used and how the categories were generated is reported in the Methodology Chapter. As the primary data instrument, it was my eyes that "saw" the emerging themes. By asking participants to regularly engage in member-checking, engaging in group conversations as a way to provide balance to conversations, and having multiple data sources, I was able to control for some of this subjectivity.

Finally, this research is based on responses given by participants during several individual conversations and two group conversations. The participants were asked to share their experiences and feelings about the mathematics courses they had taken within their programs. Although the participants were asked to answer each question honestly and frankly, they may have tried to give a response that they felt the researcher wanted to hear, instead of revealing their true feelings. Throughout the research, the assumption is made that the mathematics education doctoral students are reporting their true feelings.

Using multiple data sources such as journals and written reflections helped to control for this.

Suggestions for Further Research

This study adds to the growing research on mathematics education doctoral student preparation. Studies have been conducted to access the particular characteristics of doctoral program design, components of doctoral programs, retention, research preparation, and opportunities to work. This study adds a humanistic/experiential piece of the impact of mathematics courses on students' lives and preparation. The doctoral students who participated in this study were eager to share their voices and experiences. This was directly communicated to me and also evident in the nature of their comments and freeness to speak. With regard to future research, a similar study could be conducted with a larger sample of mathematics education doctoral students from multiple programs in order to obtain a picture of the larger landscape of mathematic education doctoral student experiences in mathematics.

This research suggests that the nature of mathematical experiences that mathematics education doctoral students have in their doctoral programs are valuable to their understanding of what it means to do mathematics in their preparations to become mathematics educators. This study could be expanded in a number of ways. Based on the findings, a proposal for further and continuing study would be to investigate if and to what degree these mathematical experiences translate and affect practice. That is, seeking to understand how the various mathematics experiences that participant had affect their own pedagogical decisions as they step into classrooms as mathematics educators.

Another avenue of potential study relates to the fact that different doctoral programs have different objectives. Some doctoral programs offer mathematics courses through different departments. Some mathematics programs are focused more on the research preparation of doctoral students versus the teaching preparation of doctoral students. Examining the mathematical courses experiences of students from programs with differing objectives would provide a context for further study.

Most of the participants acknowledged the desire for taking a mathematics course that would provide opportunities to study mathematics that would enhance their understandings of the development of k-12 mathematics content. This kind of course, was also deemed important, as it was the original description of the Foundations of Mathematics course. However, while the course may have aimed at this objective, the participants did not recognize this objective in their work. Further investigation could help identify the kind of content and course experiences that would provide mathematics education doctoral students with this desired opportunity.

A similar study conducted with preservice teachers could be beneficial for thinking about helping preservice teachers connect their mathematical experiences with the ideas presented in mathematics methods courses.

Final Thoughts

In the course of their careers, mathematics educators need broad and deep mathematical knowledge. This knowledge is necessary for the multiple roles and responsibilities that mathematics educators may be faced with including, but not limited to teaching, designing curriculum, conducting research, examining student learning or

issues of teacher knowledge, working with preservice or inservice teachers, or consulting on educational policies.

This knowledge, along with knowledge of teaching and learning, and an extensive list of research opportunities and experiences are part of the core preparation of preparing Stewards of the Discipline, as described by the Carnegie Foundation (2005).

Stewards think into the future and act on behalf of those yet to come. A steward of the discipline, then, thinks about the continuing health of the discipline and how to preserve the best of the past for those who will follow. Stewards are concerned with how to foster renewal and creativity. Perhaps most important, a steward considers how to prepare and initiate the next generations of stewards (Golde, 2006, p. 13).

If mathematics education doctoral programs are serious about preparing Stewards of the Discipline, this study provides important considerations for those responsible for the preparation of doctoral students. The mathematical experiences that mathematics education doctoral students typically have in their mathematics coursework are not sufficient in providing the kinds of experiences that experts and Stewards of the Discipline deem important for the profession. As illustrated in this study, mathematical course experiences not only directly affect doctoral students' understandings of mathematics but also provide models of mathematics teaching and learning that either continue to solidify existing experiences of traditional mathematics or provide models of mathematics classrooms that reflect the vision of reform. Having opportunities to experience mathematics, in ways similar to students' experiences in FM, provide students with models of mathematics teaching and learning that can affect the future of mathematics education.

MATHEMATICAL INTERLUDE THREE

This interlude is a crystallization that I wrote following a day of intense mathematical study at a recent visit to “math camp”. The camp, actually a week-long immersion institute for mathematics educators and mathematicians, was offered through the Center for the Scholarship of School Mathematics (<http://cssm.edc.org/>) and provided me with opportunities to return again to the *doing* of mathematics. Led by the same mathematician who co-taught the FM courses, participants were provided with time and support to engage in a piece of mathematics that we were interested in. This crystallization captures an experience at the Institute and a piece of mathematics that was important to me.

A Crystallization on Experiences Following Rectangles and Squares

It seems important that I start this crystallization with a description of a mathematical experience that occurred last night. A friend of mine is collecting data for a mathematics education research study. He asked me to be a participant. He informed me that the participation would include “taking” an algebra test (written for middle-grades algebra students). Immediately following the completion of the test, he would interview me about the answers I had provided for particular problems on the test and ask me about strategies I may have used. I agreed to take the test. I sailed through page one which consisted of problems asking me to simplify, solve, and factor. Shortly after page one of the test, I began to regret my decision to participate.

As a mathematics education doctoral student who comes to this place with a background in teaching third grade and a fairly conservative mathematics background, I am conscious of my mathematics pedigree particularly in the presence of others. I

remember the faculty member who said “How can you get a PhD in mathematics education and not know any mathematics?” always wondering if she was secretly talking about me. And so, on page two when I was completely stumped by how to solve quadratic equations that required more than ‘simple’ (I don’t use that term lightly) factoring, I began to wonder how I was getting a Ph.D. in mathematics education. I knew I would need to use the quadratic formula to solve these problems and I even had the formula in front of me. But the longer I stared at it, the less I knew what a , b , or c was. I couldn’t remember what to do with all that stuff and was clueless on how to start thinking reasonably about what to do next. It was more than the fact that I couldn’t remember how to follow the procedure, it was that I wasn’t sure I knew what “quadratic” really meant and I started to panic. I thought I should tell my friend that I didn’t want to really mess up his data so I would excuse myself from the test. Besides, I hadn’t signed the consent form yet. The remaining pages were more of the same. Some of the problems I remembered how to solve while some were so uninviting that I left them blank even after racking my brain for some way to make sense of the thing. I finally “finished” with several problems blank and several I was sure was wrong about. My friend was polite in our interview, telling me he thought some of my strategies were “unique”...

After the test, I was feeling quite insecure about my ability to know and do mathematics. Perhaps that faculty member was right. Maybe I had no business in this business. And then today I was reminded again about why my mathematical understanding is a valid and important. Although I can’t attach the following mathematics work to my recently failed algebra test, it seems important that I attempt to

capture and clarify some of the mathematical work that I did today as a way to show others, and more importantly to me, that I can *do* mathematics.

I've had a growing intrigue for numbers that just keep going and going and going and I've been fascinated by learning different ways to understand and represent these kinds of numbers. As we were "discovering" continued fractions in an exploration this week, my curiosity was becoming fertilized. On Wednesday I was mucking around with the continued fraction for $\sqrt{2}$ during our project time. With a teammate, I was able to construct the continued fraction for $\sqrt{2}$, identify its squareness sequence, find its side ratio sequence from calculating steps in the continued fraction and then continuing the ratio sequence by identifying a pattern within the sequence.

Today, I decided to try $\sqrt{3}$. My justification for "trying" this new number was to attempt to transfer what I learned in working with $\sqrt{2}$, and demonstrate a level of understanding in my pursuit to explain it to others.

.....

I start with the assumption that $\sqrt{3}$ is a little bigger than 1 but not quite as large as 2. That is $\sqrt{3}$ is equal to 1 and a little more: $\sqrt{3} = 1 + x$. Doing 'simple' algebra, I can solve for x and find that $x = (\sqrt{3} - 1)$. I can rewrite the original equation with my new value of x :

$$\sqrt{3} = 1 + (\sqrt{3} - 1)$$

This means that $\sqrt{3}$ is equal to 1 and "a little more". In this case the little more is less than 1, which I know is another way to say fraction. The form for continued fractions is a constant plus a fraction where the numerator is 1. So, another way to write that is:

$$\sqrt{3} = 1 + \frac{1}{\frac{1}{(\sqrt{3} - 1)}} \quad (\text{For clarity, I will call this stage 1})$$

But, now I am really uncomfortable because my algebra teacher always told me that we just can't have square roots in the denominators of fractions. So I will get rid of it (in honor of her). So...

$$\sqrt{3} = 1 + \frac{1}{\frac{1 \cdot (\sqrt{3} + 1)}{(\sqrt{3} - 1) \cdot (\sqrt{3} + 1)}}$$

$$\sqrt{3} = 1 + \frac{1}{\frac{(\sqrt{3} + 1)}{2}}$$

Now, based on this fraction, I can continue by asking myself, “How many 2’s are in $(\sqrt{3} + 1)$?” Based on my original assumption, I can estimate that there is one 2 in $(\sqrt{3} + 1)$ with a remainder that is the difference between 1 and $\sqrt{3}$. This difference can be written as $\sqrt{3} - 1$. My new fraction is...

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{(\sqrt{3} - 1)}{2}}$$

I know another way to write this is (again, wanting continued fraction form):

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{\frac{2}{(\sqrt{3} - 1)}}}$$

And again, I will remove the $\sqrt{}$ from the denominator...resulting in:

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{\frac{(\sqrt{3} + 1)}{1}}}$$

And I ask myself...how many 1’s are in $(\sqrt{3} + 1)$? There are two 1’s with a bit left over. How much left over? $(\sqrt{3} - 1)$ is left over. My new fraction is...

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{(\sqrt{3} - 1)}{1}}}$$

I know another way to write this is:

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{(\sqrt{3}-1)}}}$$

And now I come to a place that looks familiar: This last part of this stage looks exactly like the last part of stage 1. My conjecture is that these results are cyclic and I will continue to recycle through these stages.

Now, looking this pattern, I can see that:

$$\sqrt{3} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2}}}}}} \dots$$

Now, I can calculate the approximate values of each of these stages...(Another way I think about stage is each time I add a new + sign).

Stage 1: 1

$$\text{Stage 2: } 1 + \frac{1}{1} = 2$$

$$\text{Stage 3: } 1 + \frac{1}{1 + \frac{1}{2}} = 1\frac{2}{3} = \frac{5}{3}$$

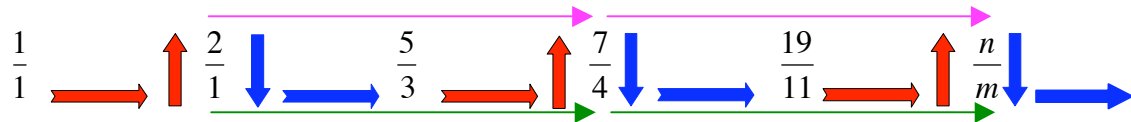
$$\text{Stage 4: } 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1}}} = 1\frac{3}{4} = \frac{7}{4}$$

$$\text{Stage 5: } 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2}}}} = 1\frac{8}{11} = \frac{19}{11}$$

When I write these ratios in order, I can see them approaching $\sqrt{3}$.

Stage	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Ratio	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{5}{3}$	$\frac{7}{4}$	$\frac{19}{11}$	$\frac{n}{m}$
Decimal	1.0	2.0	1.66	1.75	1.72	

One interesting thing that I noticed was how these decimal approximations seemed to bounce around an approximation for $\sqrt{3}$. (greater than, less than, greater than, less than...) In this vein, I would conjecture that in stage 6, the decimal approximation would be greater than the value of $\sqrt{3}$ but closer an approximation to the value than stage 5. Now I have a series of decimal approximations but I don't want to have to do the arithmetic at every stage to determine the decimal. So, I started looking for a pattern. I started with the pattern that I remembered finding in the $\sqrt{2}$. This pattern did not fit. So, after more mucking and a little help from my friends, we found a pattern that helped us produce the next term. Our pattern actually has 4 pieces....



By following similar colored arrows, I can add two previous terms to find the term “at the end of the arrow path”.

I know that the further I continue the ratio series, the closer approximation I have. With these patterns, I can calculate the subsequent ratios without having to “do all that math”. Interestingly, one of my friends introduced me to a Magic Box that provided an even easier way to find these ratios. The Magic Box is quite nice and much neater than most of all the work I just did. However, there is invaluable comfort in knowing that someday when I’m taking a test on number theory and I forget the Magic Box, I can just start drawing rectangles.

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